

MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 1: VOLUMES & SURFACES

ANSWERS

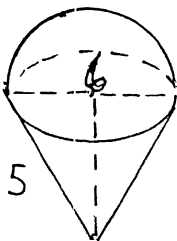
A) _____

B) _____

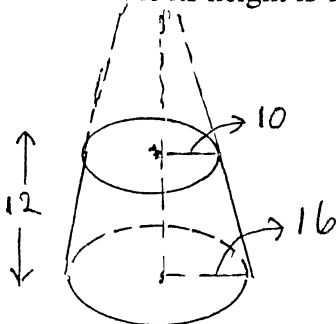
C) _____

A) A right circular cylinder of height 9 cm and diameter 8 cm has a hole of diameter 4 cm drilled out of its center*. Find the total surface area of solid remaining leaving the result in terms of pi. *=(along the axis of symmetry)

B) A right circular cone, apex down as shown, has a slant height of 5 cm and a base diameter of 6 cm. A hemisphere is sitting on top of the cone. Find the volume in terms of pi of the solid formed by the cone and the hemisphere.



C) The truncated cone shown was formed by cutting off the top of a right circular cone with a plane parallel to its base. The radii of the bases of the truncated cone are 10 cm and 16 cm while its height is 12 cm. Calculate the volume of truncated cone in terms of pi.



**MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 2: PYTHAGOREAN RELATIONS**

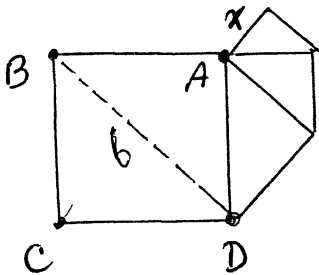
ANSWERS

A) _____

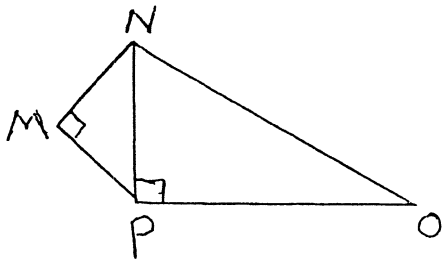
B) _____

C) _____

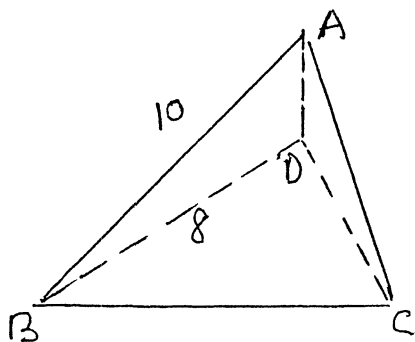
A) The diagonal of square ABCD is 6, and three 45-45-90 triangles adjoin it on the right in succession. The length of the leg labeled x is the reduced fraction a/b . Find the value of $a+b$.



B) In NOPM, $MN = MP = 7$, $\angle NMP = \angle NPO = 90^\circ$, and $\angle O = 30^\circ$. Calculate PO in simplified radical form.



C) In tetrahedron A-BCD, $\overline{AD} \perp \text{plane } BCD$, $AB = 10$, $BD = 8$, $\angle CAD = \angle DCA$, and $\angle BDC = 60^\circ$. Calculate BC in simplified radical form.



MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 3: LINEAR EQUATIONS

ANSWERS

A) _____

B) _____

C) _____

A) Solve for x , leaving the result as a ^{simplified} ~~reduced~~ fraction

$$4(x - 3) + 3(x + 7) - \frac{6}{5}(5x + 10) = \frac{2}{3}(x - 5) - \frac{1}{3}(x + 4)$$

B) The coins from Jeremy's tips total \$3.16. He has two more dimes than quarters, one less than twice as many nickels as dimes, and three more pennies than dimes. How many coins does he have?

C) Karin's exercise route is 4 miles long. She alternates running one-quarter mile at ten mph, and then walking one-half mile at five mph. How many minutes does it take her to complete her route?

MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 4: FRACTIONS & MIXED NUMBERS
NON-CALCULATOR

ANSWERS

A) _____

B) _____

C) _____

A) If $\frac{1}{a(b+1)} + \frac{1}{b(a+1)} = \frac{1}{(a+1)(b+1)}$, what is the value of $\frac{1}{a} + \frac{1}{b}$?

B) The numerator of a fraction is two less than the denominator. When both the numerator and the denominator are increased by five, the result is $\frac{4}{3}$ of the original fraction. ~~What~~ Find *all possible original fractions.*

C) If $\frac{x-3y}{x+2y} = 4\frac{2}{3}$, what is the value of $\frac{3x}{4y}$ expressed as a fraction?

MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 5: INEQUALITIES & ABSOLUTE VALUES
ANSWERS

A) _____

B) _____

C) _____

A) Solve for x : $x^3 < 5x^2 + 24x$.

B) Solve for x : $|4 - 2x| = x^2 - 3x + 2$

C) Solve for x : $\frac{1}{x^2} - \frac{5}{x} < 24$

MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 6: EVALUATIONS

ANSWERS

A) _____

B) _____

C) _____

A) If $4x + 5y + 6z + 3w = 12$, and $2x + 3y + 4z + w = 8$; find the value of $x + y + z + w$.

B) If $x + y = 7$ and $x - y = 3$, what is the value of 3^{2x-3y} ?

C) Find the value of $(234567891)(234567891) - (234567895)(234567887)$.

OCTOBER 2003
ROUND 7: TEAM QUESTIONS

ANSWERS

A) _____ D) _____

B) _____ E) _____

C) _____ F) _____

A) An ice cream cone has the shape of a square pyramid of height 5 inches where the side of the square base is 4 inches. A spherical scoop of ice cream of diameter 4 inches is placed in the cone. If the ice cream were allowed to melt, the volume of ice cream that would overflow the cone is $\frac{a - b\pi}{3}$. Find the ordered pair (a, b).

B) In rectangle ABCD, E is on diagonal AC so that $AE : AD : EC = 2 : 3 : 4$, and $BC = 39$. Calculate EB in simplified radical form.

C) Jon has a number of coins in his pocket. Eleven of them are nickels, one-seventh are dimes, and one-third are pennies. What is the total value of Jon's coins? *(He has only pennies, nickels, and dimes)*

D) Solve for x: $\left(\frac{x+3}{x-1}\right)^2 - 6 = \frac{x+3}{x-1}$

E) Solve for x: $\left(\frac{x+1}{x-2}\right)^2 - \frac{x+1}{x-2} - 2 < 0$

F) Given $x * y = 2x + 3y$, solve the equation $(3 * a) * (a * 3) = 24 * 13$ for a.

MML: OCTOBER 2003 Contest #1

Round 1: Geometry: Volume/Surface Area

- A) $132\pi \text{ cm}^2$
- B) ~~24 cm~~ $30\pi \text{ cm}^3$
- C) $2064\pi \text{ cm}^3$

Round 2: Pythagorean Theorem

- A) 5
- B) $7\sqrt{6}$
- C) $2\sqrt{13}$

Round 3: Linear Equations

- A) $-5/2$
- B) 40
- C) 39

Round 4: Fractions and Mixed Numbers

- A) -1
- B) $3/5$ and $-10/-8$ (may not be reduced)
- C) $-111/44$

Round 5: Inequalities and Absolute Value

- A) $x < -3$ or $0 < x < 8$
- B) -1, 2, 3
- C) $x < -1/3$ or $x > 1/8$

Round 6: Evaluations

- A) 2
- B) 81
- C) 16

Round 7: Team

- A) ~~(-80, 38)~~ -32
- B) $13\sqrt{13}$
- C) 92¢
- D) 3, $-1/3$
- E) $x < 1/2$ or $x > 5$
- F) 4

MASSACHUSETTS MATHEMATICS LEAGUE
 OCTOBER 2003
 ROUND 1 VOLUMES & SURFACES

ANSWERS

A) 132π

B) 30π

C) 2064π

A) A cylindrical can of radius 4 cm and height 4 cm is drilled out of its center. Find the total surface area of solid remaining leaving the result in terms of π.



$$R = 4 \quad h = 4$$

$$2\pi r h = 2\pi(4)(4) = 32\pi \quad \text{top surface} = \pi R^2 = 16\pi$$

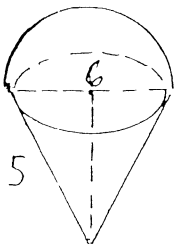
$$2\pi r^2 = 2\pi(4)^2 = 32\pi \quad \text{Total surface area}$$

$$10\pi + 16\pi$$

$$2\pi R^2 - 2\pi r^2 = 2\pi(16) - 2\pi(4) = 22\pi - 8\pi = 14\pi$$

$$\underline{\text{Ans. } 108\pi + 24\pi = 132\pi}$$

B) A right circular cone (apex down) as shown has a slant height of 5 cm and a base diameter of 6 cm. A hemisphere is sitting on top of the cone. Find the volume in terms of π of the solid formed by the cone and the hemisphere.



$$V_S = \frac{4}{3}\pi r^3 \quad V_C = \frac{1}{3}\pi r^2 h$$

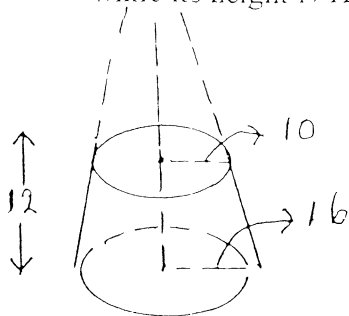
$$V_H = \frac{1}{3}\pi \cdot 3^2 \cdot 4 = 12\pi$$

$$r = 3, \quad h = \sqrt{5^2 - 3^2} = 4$$

$$V_S = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi$$

$$\underline{\text{ANS } 30\pi}$$

C) The truncated cone shown was formed by cutting off the top of a right circular cone with a plane parallel to its base. The radii of the bases of the truncated cone are 10 cm and 16 cm while its height is 12 cm. Calculate the volume of truncated cone in terms of π.



$$V = \frac{1}{3}\pi h (r_1^2 + r_1 r_2 + r_2^2)$$

$$= \frac{1}{3} 12\pi (100 + 160 + 256) = 4\pi \cdot 516$$

$$= 2064\pi$$

MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 2: PYTHAGOREAN RELATIONS

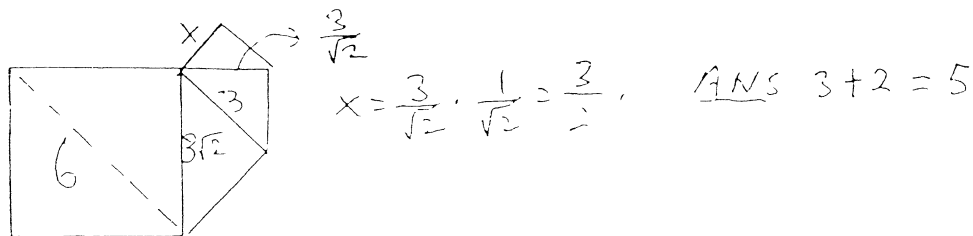
ANSWERS

A) 5

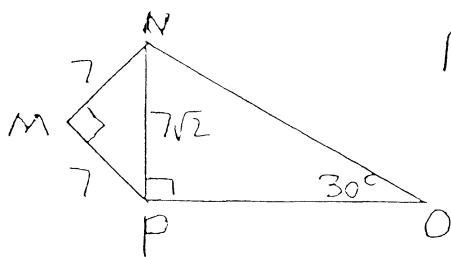
B) $7\sqrt{6}$

C) $2\sqrt{13}$

A) The diagonal of square ABCD is 6, and three 45-45-90 triangles adjoin it on the right in succession. The length of the leg labeled x is the reduced fraction a/b. Find the value of a+b.

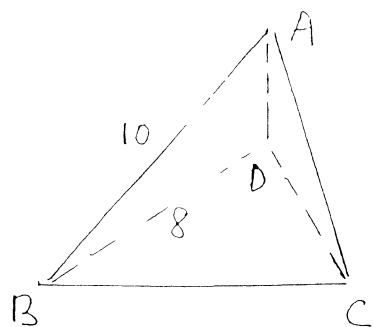


B) In NOPM, $MN = MP = 7$, $\angle NMP = \angle NPO = 90^\circ$, and $\angle O = 30^\circ$. Calculate PO in simplified radical form.



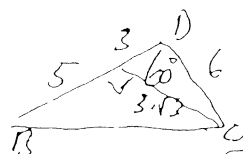
$$PO = 7\sqrt{2} \cdot \sqrt{3} = 7\sqrt{6}$$

C) In tetrahedron A-BCD, $\overline{AD} \perp \text{plane } BCD$, $AB = 10$, $BD = 8$, $\angle CAD = \angle DCA$, and $\angle BDC = 60^\circ$. Calculate BC in simplified radical form.



$$AD = \sqrt{10^2 - 8^2} = 6$$

$$DC = AD = 6$$



$$BC = \sqrt{5^2 + 3\sqrt{3}^2} = \sqrt{25 + 27} = \sqrt{52}$$

MASSACHUSETTS MATHEMATICS LEAGUE
 OCTOBER 2003
 ROUND 3: LINEAR EQUATIONS

ANSWERS

A) $-\frac{5}{2}$

B) 40

C) 39

A) Solve for x , leaving the result as a reduced fraction

$$4(x-3) \cdot 3(x+7) - \frac{6}{5}(5x+10) = \frac{2}{3}(x-5) - \frac{1}{3}(x-4)$$

$$4/x - 12 + 3x + 21 - 6x - 12 = \frac{2}{3}x - \frac{10}{3} - \frac{1}{3}x - \frac{4}{3}$$

$$x - 3 = \frac{1}{3}x - \frac{14}{3}$$

$$\frac{2}{3}x = 3 - \frac{14}{3} = -\frac{5}{3}$$

$$x = -\frac{5}{2}$$

B) The coins from Jeremy's tips total \$3.16. He has two more dimes than quarters, one less than twice as many nickels as dimes, and three more pennies than dimes. How many coins does he have?

$$D = \# \text{ dimes} \quad 25(D-2) + 10D + 5(2D-1) + (D+3) = 316$$

$$Q = D-2 \quad 25D - 50 + 10D + 10D - 5 + D + 3 = 316$$

$$N = 2D-1 \quad 46D - 52 = 316 \quad Q = 6$$

$$P = D+3$$

$$46D = 368$$

$$N = 15 \quad \underline{\text{ANS}} \quad 40$$

$$D = 8$$

$$P = 11$$

C) Karin's exercise route is 4 miles long. She alternates running one-quarter mile at ten mph, and then walking one-half mile at five mph. How many minutes does it take her to complete her route?

$$\begin{array}{cccccccccccc} R & W & R & W & R & W & W & R & W & R & W & R \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{array}$$

$$\frac{1}{4} \cdot 6 = \frac{3}{2} \text{ mi, running}$$

$$\frac{1}{2} \cdot 5 = \frac{5}{2} \text{ mi, walking}$$

$$\frac{3}{2} \cdot \frac{1}{10} \cdot 60 + \frac{5}{2} \cdot \frac{1}{5} \cdot 60 = \text{Time in min}$$

$$9 + 30 = \underline{39}$$

MASSACHUSETTS MATHEMATICS LEAGUE
 OCTOBER 2003
 ROUND 4: FRACTIONS & MIXED NUMBERS
 NON-CALCULATOR

ANSWERS

A) -1

B) $-10/-8, 3/5$

C) $-111/44$

A) If $\frac{1}{a(b-1)} + \frac{1}{b(a-1)} = \frac{1}{(a-1)(b+1)}$, what is the value of $\frac{1}{a} + \frac{1}{b}$?

$$\frac{1}{a(b+1)} + \frac{1}{b(a+1)} = \frac{1}{ab}$$

$$\text{So } \frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab} = -1$$

$$-b + 1 + ab + a = \frac{1}{b}$$

$$1 + a = -ba$$

$$\frac{b+a}{ba} = -1$$

B) The numerator of a fraction is two less than the denominator. When both the numerator and the denominator are increased by five, the result is $4/3$ of the original fraction. What was the original fraction?

$$\frac{x-2}{x} = \text{orig fract} \quad \frac{x+3}{x+5} = \frac{4(x-2)}{3x} \quad x^2 + 3x - 40 = 0$$

$$(x+8)(x-5) = 0$$

$$3x(x+3) = (4x-8)(x+5) \quad x = -8, x = 5$$

$$3x^2 + 9x = 4x^2 + 12x - 40 \quad \text{ANS } \frac{-10}{-8}, \frac{3}{5}$$

C) If $\frac{x}{x+2y} = \frac{3}{4}$ and $\frac{y}{x-3y} = \frac{2}{3}$, what is the value of $\frac{3x}{4y}$ expressed as a fraction?

$$\frac{x}{x+2y} = \frac{3}{4}$$

$$3x - 9y = 14x + 28y$$

$$-11x = 37y$$

$$\frac{x}{y} = -\frac{37}{11}, \quad \frac{3x}{4y} = -\frac{111}{44}$$

MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 5: INEQUALITIES & ABSOLUTE VALUES

ANSWERS

A) $x < -3, 0 < x < 3$

B) $-1, 2, 3$

C) $x > \frac{1}{8}, x < -\frac{1}{3}$

write it as $x^3 - 5x^2 - 24x < 0$

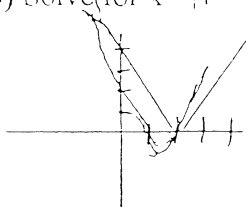
$$x^3 - 5x^2 - 24x < 0$$

$$x(x+3)(x-8) < 0$$



ANS $x < -3, 0 < x < 3$

B) Solve for x $|4 - 2x| = x^2 - 3x + 2$



$$4 - 2x = x^2 - 3x + 2$$

$$x^2 - x - 2 = 0$$

$$(x+1)(x-2) = 0$$

$$x = -1, x = 2$$

no intersection for $x > 2$

$$\text{if } x > 2$$

$$2x - 4 = x^2 - 3x + 2$$

$$0 = x^2 - 5x + 6$$

$$x = 2 \text{ or } 3$$

$$\therefore x = 3$$

C) Solve for x $\frac{1}{x^2} - \frac{5}{x} < 24$

$$\frac{1}{x^2} - \frac{5}{x} - 24 < 0$$

$$-24x^2 - 5x + 1 < 0$$

$$24x^2 + 5x - 1 > 0$$

$$(8x - 1)(3x + 1) > 0$$

ANS $x > \frac{1}{8}, x < -\frac{1}{3}$

MASSACHUSETTS MATHEMATICS LEAGUE
 OCTOBER 2003
 ROUND 6: EVALUATIONS

ANSWERS

A) 2

B) 81

C) 16

A) If $x + y = 7$ and $x - y = 3$ what is the value of $3^{2x - 3y}$?

$$2x + 3y + 4z + 6w = 8^2$$

$$2x + 2y + 2z + 2w = 4$$

$$x + y + z + w = 2$$

B) If $x + y = 7$ and $x - y = 3$ what is the value of $3^{2x - 3y}$?

$$x - y = 3$$

$$3^{2(5) - 3(2)} = 3^{10 - 6} = 3^4 = 81$$

$$2x = 10$$

$$x = 5$$

$$2y = 4$$

$$y = 2$$

C) Find the value of $(234567891)(234567891) - (234567895)(234567887)$

$$(x)(x) - (x+4)(x-4)$$

$$x^2 - (x^2 - 16) = 16$$

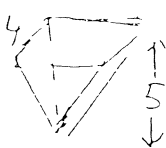
A) $(-80, -32)$ D) $3, -1/3$

B) $13\sqrt{13}$ E) $x < 1/2, x > 5$

C) 92 cents F) 4

A) An ice cream cone has the shape of a square pyramid of height 5 inches where the side of the square base is 4 inches. A spherical scoop of ice cream of diameter 4 inches is placed in the cone. If the ice cream were allowed to melt, the volume of ice cream that would overflow the cone is

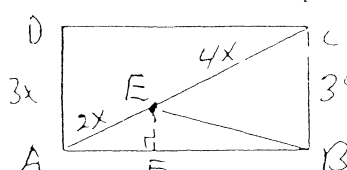
$V = \frac{1}{3}bh$ for the cone (to be)



$V_P = \frac{1}{3}(16)(5) = \frac{80}{3}$ $V_S = \frac{4}{3}\pi(2^3) = \frac{32\pi}{3}$

ANS $(-80, -32)$

B) In rectangle ABCD, E is on diagonal AC so that $AF = AD = FC = 2$, $3 = 4$ and $BC = 30$. Calculate EB in simplified radical form.



$x = 13$ $AE = 2x = 26$, $EC = 52$, $AC = 78$

$\triangle ABC = 30^\circ, 60^\circ, 90^\circ$ so $\angle CAB = 30^\circ$

$AB = 39\sqrt{3}$, $AF = 13\sqrt{3}$, $FB = 26\sqrt{3}$, $EF = 13$

$EB = 13\sqrt{1^2 + 2\sqrt{3}^2} = 13\sqrt{13}$

C) Jon has a number of coins in his pocket. Eleven of them are nickels, one-seventh are dimes, and one-third are pennies. What is the total value of Jon's coins?

$x = \# \text{Coins}$ $\frac{1}{7}x + \frac{1}{3}x + 11 = x$, $\frac{10x}{21} + 11 = x$, $\frac{11x}{21} = 11$, $x = 21$

Nickels - 55¢, Dimes - 30¢, Pennies - 7¢ Ans 92 cents

D) Solve for x $\left(\frac{x+3}{x-1}\right)^2 - 6 = \frac{x-3}{x-1}$ $\frac{x+3}{x-1} = 3$, $x+3 = 3x-3$, $2x = 6$, $x = 3$

$\left(\frac{x+3}{x-1} - 3\right)\left(\frac{x+3}{x-1} + 2\right) = 0$ $\frac{x+3}{x-1} = -2$, $x+3 = -2x+2$, $3x = -1$, $x = -\frac{1}{3}$

E) Solve for x $\left(\frac{x}{x-2} - \frac{1}{2}\right) \cdot \frac{x}{x-2} - 2 = 0$ $(x+1) - (x+1)(x-2) - 2(x-2) < 0$

multi by $(x-2)^2$ To get $-2x^2 + 11x - 5 < 0$

Note! $(x-2)^2 > 0$ $2x^2 - 11x + 5 > 0$ $(2x-1)(x-5) > 0$

Ans $x < 1/2$ or $x > 5$

F) Given $x * y = 2x + 3y$, solve the equation $(3 * a) * (a * 3) = 24 * 13$ for a

$2(6 + 3a) + 3(2a + 9) = 48 + 39 = 87$, $12a + 39 = 87$, $a = 4$