MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 1: TRIG. TRIANGLES

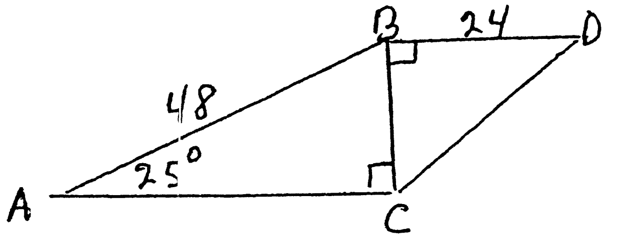
ANSWERS

A)______________

B)______________

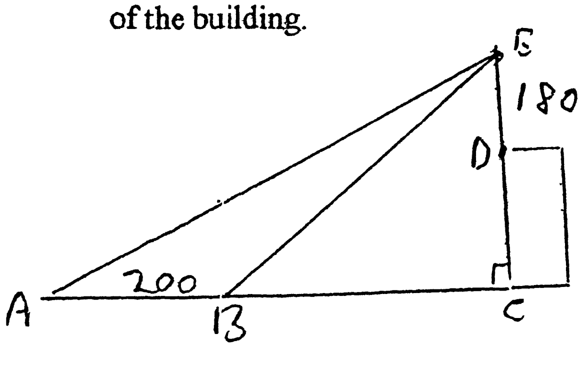
C)______________

A) Given ΔABC, $\angle A = 25^0$, $\angle C = 90^0$, $AB = 48$. In ΔBCD, $\angle DBC = 90^0$, and BD = 24. To the nearest tenth, calculate the degree measure of $\angle BDC$.



B) A regular pentagon is inscribed in a circle of radius 30 inches. Calculate to the nearest tenth of an inch the length of a diagonal of the pentagon.

C) A 180 foot tall antenna is located on top of a building. Some distance from the building the angle of elevation of the top of the antenna is 64.3 degrees. From a point 200 feet farther from the building, the angle of elevation is 53.4 degrees. To the nearest integer, calculate the height of the building.



MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 2: NUMBER THEORY

ANSWERS

A)_____________

B)_____________

C)_____________

A) During rush hour on December fourth, 20% of the cars on Route 314 took exit 1, 25% of those remaining took exit 2, and 10% remaining after that took exit 3. If 162 cars continued on Route 314, how many cars traveled the route during rush hour that day?

B) A palindrome reads the same from right to left or vice versa. For example, 37673 is a palindrome. How many palindromes are there between 10,000 and 20,000?

C) The length of each side of a triangle is a prime, and its perimeter is also a prime. What is the smallest possible perimeter that the triangle could have?

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 3: ANALYTIC GEOMETRY

ANSWERS

A)______________

B)______________

C)______________

A) If A = (-3, 5), B = (7, 1), and C = (5, 9), calculate in standard ax + by = c form, the equation of the midline of triangle ABC which is parallel to $\overline{AB}$.

B) If A = (5, -6), B = (1, 2), and C = (k, k); calculate the value of k so that $\overline{AB} \perp \overline{AC}$.

C) Calculate the area of the region enclosed by the graph of the equation $2|x-3| + 3|y+5| = 12$.

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 4: LOGS & EXPONENTIALS
NON-CALCULATOR

ANSWERS

A)_______________

B)_______________

C)_______________

A) Solve for x: $\log_9 x = \log_{16} 320 - \log_{16} 5$

B) Solve for x: $\left(\frac{1}{4}\right)^{x - x^2} = 8^{2-x}$

C) Solve for x: $\log_2(-2x-1) - \log_{\sqrt{2}} 2 + \log_2(-x+3) = 0$

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 5: RATIO & PROPORTION

ANSWERS

A)____________

B)____________

C)____________

A) Mr. Allen and Mr. Baker are business partners, their profits being divided in the ratio of 4 to 5 respectively. Mr. Currier and Mrs. Dodge are also in business, their profits being divided in the ratio of 4 to 7 respectively. When the two businesses merged, Mr. Allen and Mr. Baker's business put in \$9 for every \$11 put in by Mr. Currier and Mrs. Dodge's business. When the new business had a profit of \$12,000, how much should Mrs. Dodge receive?

B) If the ratio of a to b is 4 to 9, the ratio of b to c is 12 to 7, and the ratio of a to d is 4 to 7; calculate the ratio of c to d as a reduced fraction.

C) The height of a cylindrical tank varies directly as the volume and inversely as the square of the radius. If the height is 24, the volume is 7536, and the radius is 10; find the height when the volume is 628 and the radius is 5.

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 6: POLYGONS

ANSWERS

A)______________

B)______________

C)______________

A) Calculate the number of diagonals that can be drawn from a single vertex in a regular polygon whose interior angles each measure 165 degrees.

B) A pennant is designed in the shape of an isosceles triangle, triangle ABC with vertex angle B Points P and R are located on segments CB and AB respectively so that AC = AP = PR = RB. Calculate the measure of angle B.

C) A yard in the shape of a right triangle has sides that measure 60, 80, and 100 feet. A fence runs from the vertex of the right angle to the hypotenuse separating the yard into two regions of equal perimeter In simple radical form, calculate the length of the fence.

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 7: TEAM QUESTIONS

ANSWERS

A)____________ D)____________

B)____________ E)____________

C)____________ F)____________

A) A submarine submerging at an angle of 5 degrees is propelled at the rate of 9 mph. Calculate to the nearest tenth how many seconds it would take the submarine to reach a depth of 80 feet.

B) After playing four rounds of Flip-A-Card, Al and Bob each have 16 cards. In each round of this game, the loser gives the winner enough cards from his own to double the number of cards in the winner's hand. Al lost the first two rounds, but won the last two. How many cards did Al start with?

C) If $6^p = 3$, calculate the value of $(\log_p 36^p)(\log_3 p)$.

D) Determine both points on the line $y = 3x - 2$ which are $\sqrt{5}$ units from the line $2x + y = 3$.

E) In a rectangle, the ratio of a diagonal to the perimeter is 5 to 14 while the area is 432. Calculate the length of diagonal.

F) The smallest interior angle of a certain polygon is 120 degrees, and successive angles increase by 5 degrees. That is, the angles are 120, 125, 130, etc. Calculate the number of sides of the polygon

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 1: TRIG. TRIANGLES

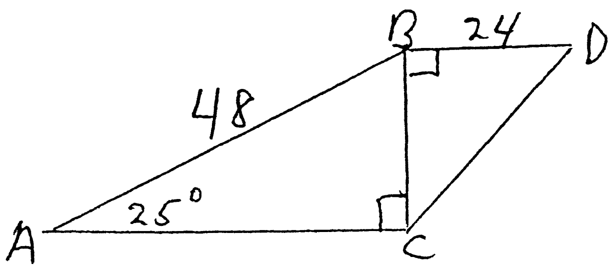
ANSWERS

A) 40.2°

B) 57.1

C) 585

A) Given ΔABC, $\angle A = 25^0$, $\angle C = 90^0$, $AB = 48$. In ΔBCD, $\angle DBC = 90^0$, and BD = 24. To the nearest tenth, calculate the degree measure of $\angle BDC$.

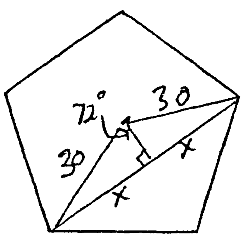


$BC = 48 \sin 25^\circ$

$\tan D = \dfrac{48 \sin 25^\circ}{24} = 2 \sin 25^\circ$

$D = \tan^{-1}(2 \sin 25^\circ) = 40.2^\circ$

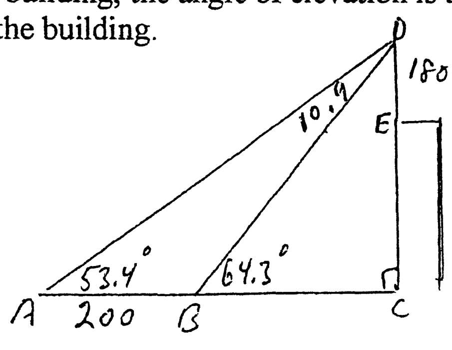
B) A regular pentagon is inscribed in a circle of radius 30 inches. Calculate to the nearest tenth of an inch the length of a diagonal of the pentagon.



$x = 30 \sin 72^\circ$

$d = 2x = 60 \sin 72^\circ = 57.1$

C) A 180 foot tall antenna is located on top of a building. Some distance from the building the angle of elevation of the top of the antenna is 64.3 degrees. From a point 200 feet farther from the building, the angle of elevation is 53.4 degrees. To the nearest integer, calculate the height of the building.



$\dfrac{BD}{\sin 53.4^\circ} = \dfrac{200}{\sin 10.9^\circ}$

$CD = BD \sin 64.3 = \dfrac{200 \sin 53.4^\circ \sin 64.3^\circ}{\sin 10.9^\circ}$

$= 765.1 \quad CE = 765 - 180 = 585$

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 2: NUMBER THEORY

ANSWERS

A) 300

B) 100

C) 7

A) On a particular section of Route 314 there were three exits. During rush hour on December fourth, 20% of the cars on Route 314 took exit 1, 25% of those remaining took exit 2, and 10% remaining after that took exit 3. If 162 cars continued on Route 314, how many cars traveled the route during rush hour that day?

$X(.80)(.75)(.90) = 162 \quad X = 300$

B) A palindrome reads the same from right to left or vice versa. For example, 37673 is a palindrome. How many palindromes are there between 10,000 and 20,000?

Must be of the form 1ABA1   ANS $10 \cdot 10 = 100$

C) The length of each side of a triangle is a prime, and its perimeter is also a prime. What is the smallest possible perimeter that the triangle could have?

2, 2, 3 $P = 7$

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 3: ANALYTIC GEOMETRY

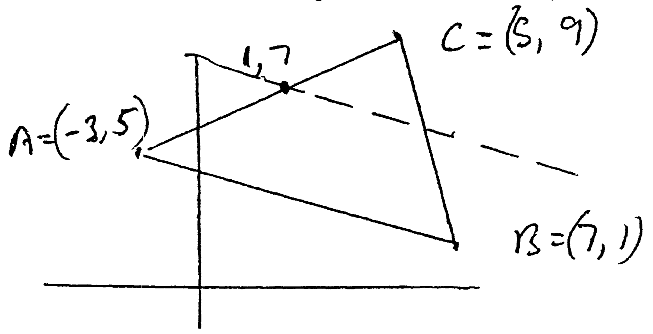
ANSWERS

A) $2x + 5y = 37$

B) -17

C) 48

A) If A = (-3, 5), B = (7, 1), and C = (5, 9), calculate in standard ax + by = c form, the equation of the midline of triangle ABC which is parallel to $\overline{AB}$.



$m_{AB} = \frac{1-5}{7+3} = -\frac{4}{10} = -\frac{2}{5}$

midline is $2x + 5y = 2 + 35 = 37$

B) If A = (5, -6), B = (1, 2), and C = (k, k); calculate the value of k so that $\overline{AB} \perp \overline{AC}$.

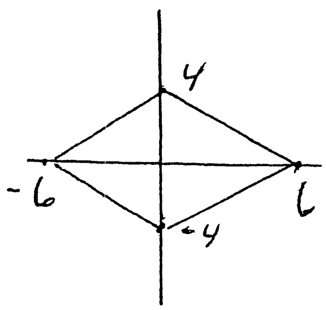
$m_{AB} = \frac{-6-2}{5-1} = \frac{-8}{4} = -2, \quad m_{AC} = \frac{k+6}{k-5}$

$\frac{k+6}{k-5} = \frac{1}{2}, \quad 2k + 12 = k - 5$

$k = -17$

C) Calculate the area of the region enclosed by the graph of the equation $2|x - 3| + 3|y + 5| = 12$.

Translate to the origin to get $2|x| + 3|y| = 12$



Area $= 4 \cdot 12 = 48$

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 4: LOGS & EXPONENTIALS
NON-CALCULATOR

ANSWERS

A) 27

B) 3/2, -2

C) -1

A) Solve for x: $\log_9 x = \log_{16} 320 - \log_{16} 5$

$$\log_9 x = \log_{16} \frac{320}{5} = \log_{16} 64 = \frac{3}{2}$$

$$x = 9^{3/2} = 27$$

B) Solve for x: $\left(\frac{1}{4}\right)^{x-x^2} = 8^{2-x}$

$$2^{-2(x-x^2)} = 2^{3(2-x)}$$

$$-2x + 2x^2 = 6 - 3x$$

$$2x^2 + x - 6 = 0$$

$$(2x-3)(x+2) = 0$$

$$x = 3/2, -2$$

C) Solve for x: $\log_2(-2x-1) - \log_{\sqrt{2}} 2 + \log_2(-x+3) = 0$

$$\log_2(-2x-1)(-x+3) = \log_{\sqrt{2}} 2 = 2$$

$$2x^2 - 6x + x - 3 = 2^2 = 4$$

$$2x^2 - 5x - 7 = 0 \quad (2x-7)(x+1) = 0$$

$x = \frac{7}{2}, -1$, only -1 checks.

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 5: RATIO & PROPORTION

ANSWERS

A) \$4200

B) 3/4

C) 8

A) Mr. Allen and Mr. Baker were business partners, their profits being divided in the ratio of 4 to 5 respectively. Mr. Currier and Mrs. Dodge were also in business, their protits divided in the ratio of 4 to 7 respectively When the two businesses merged, the first two put in \$9 for every \$11 put in by the second two. When the new business had a gain of \$12.000. how much should Mrs. Dodge get?

A's share $= \frac{4}{9}$, B's $= \frac{5}{9}$

C's $= \frac{4}{11}$, D's $= \frac{7}{11}$

in the new business A+B's share is $\frac{9}{20}$, C+D's $= \frac{11}{20}$

mrs Dodge should get $\frac{7}{11} \cdot \frac{11}{20} \cdot 12000 = 7(600) = \$4200$

B) If the ratio of a to b is 4 to 9, the ratio of b to c is 12 to 7, and the ratio of a to d is 4 to 7; calculate the ratio of c to d as a reduced fraction.

$$\frac{\frac{a}{d}}{\frac{b}{c}} = \frac{\frac{4}{7}}{\frac{12}{7}} = \frac{1}{3} = \frac{a}{d} \cdot \frac{c}{b} = \frac{c}{d} \cdot \frac{a}{b}, \text{ so } \frac{c}{d} \cdot \frac{4}{9} = \frac{1}{3}$$

$$\frac{c}{d} = \frac{1}{3} \cdot \frac{9}{4} = \frac{3}{4}$$

C) The height of a cylindrical tank varies directly as the volume and inversely as the square of the radius. If the height is 24, the volume is 7536, and the radius is 10; find the height when the volume is 628 and the radius is 5.

$$\frac{hr^2}{V} \qquad \frac{25h}{628} = \frac{100 \cdot 24}{7536}, \quad h = \frac{\cancel{100}^{\,4} \cdot 24 \cdot \cancel{628}^{\,1}}{\cancel{25}_{\,1} \cdot \cancel{7536}_{\,12}} = 4 \cdot 2 = 8$$

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 6: POLYGONS

ANSWERS

A) 21

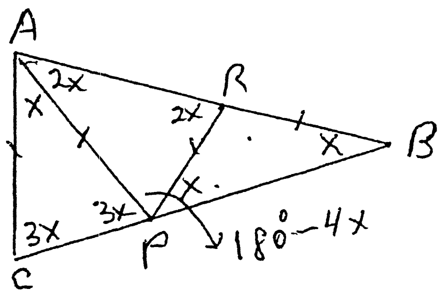
B) 180/7

C) $24\sqrt{5}$

A) Calculate the number of diagonals that can be drawn from a single vertex in a regular polygon whose interior angles each measure 165 degrees.

$n = \frac{360}{180-165} = \frac{360}{15} = 24$. The number of diagonals from a single vertex is $n-3 = 21$.

B) A pennant is designed in the shape of an isosceles triangle, triangle ABC with vertex angle B. Points P and R are located on segments CB and AB respectively so that AC = AP = PR = RB. ~~To the nearest tenth,~~ Calculate the measure of angle B.

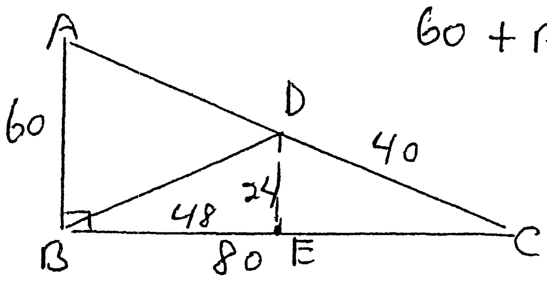


$3x + 3x + x = 180°$

$7x = 180°$

$x = \frac{180°}{7} = 25.7°$

C) A yard in the shape of a right triangle has sides that measure 60, 80, and 100 feet. A fence runs from the right angle to the hypotenuse separating the yard into two regions of equal perimeter. In simple radical form, calculate the length of the fence.



$60 + BD + (100 - CD) = 80 + CD + BD$

$160 - CD = 80 + CD$

$80 = 2CD$, $CD = 40$

$\triangle CDE \sim \triangle CAB$ so $EC = 32$ and $DE = 24$, so $BE = 80 - 32 = 48$

$BD = 24\sqrt{5}$

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 7: TEAM QUESTIONS

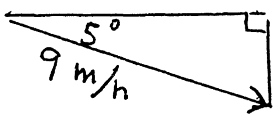
ANSWERS

A) 69.5 D) (2, 4), (0, -2)

B) 25 E) 30

C) 2 F) 9

A) A submarine submerging at an angle of 5 degrees is propelled at the rate of 9 mph. Calculate to the nearest tenth how many seconds it would take the submarine to reach a depth of 80 feet



x m/h $\quad x = 9\sin 5^\circ \frac{mi}{hr} = \frac{9\sin 5^\circ (5280)}{3600} \frac{ft}{sec}$

$= 1.1505 \frac{ft}{sec}, \quad T = 80/1.1505 = 69.5$ sec

B) Al and Bob each have 16 cards after playing four rounds of Flip-A-Card. In each round of this game, the loser gives the winner enough cards from his own to double the number of cards in the winner's hand. Al lost the first two rounds, but won the last two. How many cards did Al start with?

Al	Bob		Al	Bob
16	16		18	14
8	24		25	7
4	28			

C) If $6^p = 3$, calculate the value of $(\log_p 36^p)(\log_3 p)$.

$\log_p 36^p = \log_p 6^{2p} = 2\log_p(6^p) = 2\log_p 3$ so $2\log_p 3 \cdot \log_3 p = 2 \cdot 1 = 2$

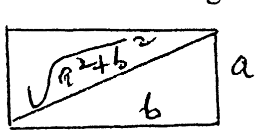
D) Determine both points on the line y = 3x – 2 which are $\sqrt{5}$ units from the line 2x + y = 3

$2x + y = c$
$2x + y = 3$
$d = \frac{|c-3|}{\sqrt{2^2+1^2}} = \sqrt{5}, \quad |c-3| = 5, \quad c = -2, 8$

$2x + y = 8$
$3x - y = 2$
$(2, 4)$

$2x + y = -2$
$3x - y = 2$
$(0, -2)$

E) In a rectangle, the ratio of a diagonal to the perimeter is 5 to 14 while the area is 432 Calculate the length of diagonal.



$\frac{\sqrt{a^2+b^2}}{2a+2b} = \frac{5}{14}, \quad 14\sqrt{a^2+b^2} = 10a + 10b$

$49a^2 + 49b^2 = 25a^2 + 50ab + 25b^2, \quad 24(a^2+b^2) = 50(432)$

$a^2 + b^2 = 900$

F) The smallest interior angle of a certain polygon is 120 degrees, and successive angles increase by 5 degrees. That is, the angles are 120, 125, 130, etc. Calculate both possibilities for the number of sides of the polygon.

$120 + 125 + \ldots + (5n + 115) = 180(n-2), \quad \frac{n}{2}(5n + 235) = 180n - 360$

$n(n + 47) = 72n - 144, \quad n^2 - 25n + 144 = 0 \quad (n-9)(n-16) = 0$

16 is extraneous.