MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 1: ANALYTIC GEOMETRY

ANSWERS

A)_____

B)_____

C)_____

A) Find the equation of the line of centers of the circles $x^2 + y^2 + 6x - 2y + 1 = 0$, and $2x^2 + 2y^2 - 8x + 12y - 24 = 0$. Write the equation in ax + by = c form.

B) A triangle with area 18 is formed by the axes and a line with slope $\frac{2}{3}$ which has a positive y-intercept. Calculate in simple radical form, the value of this positive y-intercept.

C) Find the equation of the circle with center at the origin which is tangent to the line $2x + 3y = 39$.

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 2: FACTORING & APPLICATIONS
ANSWERS

A)_____

B)_____

C)_____

A) The base of a triangle is five more than twice the altitude to that base. If the area of the triangle is 84, calculate the length of the base.

B) Find three consecutive odd integers such that the product of the first and the third added to the sum of all three is 234.

C) Factor: $2x^5 - 3x^4 - 16x^2 + 24x$

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 3: TRIG EQUATIONS
NON-CALCULATOR

ANSWERS

A)_____

B)_____

C)_____

A) Solve for $0^0 \le \theta < 360^0$, $\sin\theta + \csc\theta = 2.5$

B) Solve for $0 \le x < 2\pi$, $\dfrac{2\sin^2 x}{1-\cos x} = 1$

C) Solve for $0 \le x < 2\pi$, $\tan 2x = -3\cot x$

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 4: QUADRATICS

ANSWERS

A)_____

B)_____

C)_____

A) For what values of k will the equation $2x^2 - kx + 8 = 0$ have two equal real roots?

B) The area of a square piece of tin is 625 sq. in. Squares of equal size are cut out of the two top corners. Larger squares, each four times the area of a top corner square, are cut out of the two bottom corners. Calculate the perimeter of the resulting figure if its area is 535 sq. in.

C) If one root of $ax^2 + bx + c = 0$ is x = -2, b + c = 0, and a + b = 7; find the value of b.

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 5: SIMILAR POLYGONS

ANSWERS

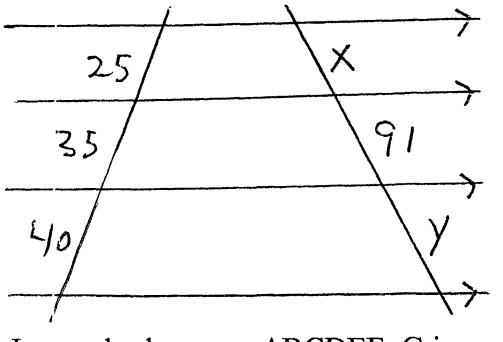
A)_____

B)_____

C)_____

A) There are two solid cubes made of the same material where the edge of one cube is three times the edge of the other. If the smaller cube weighs 2.3 grams, calculate to the nearest tenth, the weight of the larger cube.

B) In the figure shown, lines k, l, m, and n are parallel, with transversal segment lengths given. Calculate the sum of the lengths of segments x and y.



C) In regular hexagon ABCDEF, G is on $\overline{FC}$ so that $\angle CBG = 45^0$. Calculate in simple radical form, the ratio of $\overline{GC}$ *to* $\overline{CB}$.

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 6: ALGEBRA I ANYTHING

ANSWERS

A)_____

B)_____

C)_____

A) Jeff and his wife Brenda can each weed the garden in four hours. One day after they had worked together weeding for one hour, their son Kyle helped them finish the work in one-half hour. How long would it have taken Kyle to weed the garden by himself?

B) Simplify: $\dfrac{15x^2 + 11x - 12}{25x^2 - 9} \div \dfrac{3x^2 + 13x + 12}{10x^2 + 11x + 3}$

C) Solve for x: $|x - 2| + |x + 4| = 8$

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 7: TEAM QUESTIONS

ANSWERS

A)_____ D)_____

B)_____ E)_____

C)_____ F)_____

A) A parabola which has vertex V at the focus of the parabola $x^2 = 8y$, and focus at the origin O, intersects $x^2 = 8y$ at points A and B. Calculate the area of quadrilateral AVBO.

B) A two foot by three foot poster is framed with a border of uniform width. If the area of the border is the same as the area of the poster, calculate in inches the width of the border.

C) Solve for $0^0 \le \theta < 360^0$, $\cos 3\theta + \cos\theta = \sqrt{2}\cos 2\theta$

D) In the equation $ax^2 + bx + c = 0$, a, b, and c are relatively prime integers. If the product of its roots is $-\frac{8}{3}$, and the difference of its roots is $\frac{10}{3}$, calculate two possible values for b.

E) In ΔABC, AB = CB, $\angle B = 108^0$, D is on $\overline{AC}$ so that $\angle CBD$ *is twice* $\angle ABD$. If DA = 2, calculate CB in simplified radical form.

F) Mary's speed on her bike was 6 mph on the level, 4 mph downhill, and 12 mph uphill. One day she rode to Greg's house and returned by the same route in one hour. How far in miles is it to Greg's house?

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 1: ANALYTIC GEOMETRY

ANSWERS

A) $4x + 5y = -7$

B) $2\sqrt{6}$

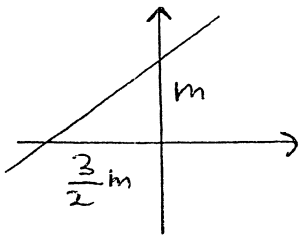
C) $x^2 + y^2 = 117$

A) Find the equation of the line of centers of the circles $x^2 + y^2 + 6x - 2y + 1 = 0$, and $2x^2 + 2y^2 - 8x + 12y - 24 = 0$. Write the equation in ax + by = c form.

$C_1 = (-3, 1)$; $C_2 = (2, -3)$. $m = \frac{-3-1}{2+3} = -\frac{4}{5}$

$4x + 5y = -12 + 5 = -7$

B) A triangle with area 18 is formed by the axes and a line with slope $\frac{2}{3}$ which has a positive y-intercept. Calculate in simple radical form, the value of this positive y-intercept.



$A = \frac{1}{2} \cdot \frac{3}{2} m \cdot m = 18$

$\frac{1}{4} m^2 = 6, \quad m^2 = 24$

$m = 2\sqrt{6}$

C) Find the equation of the circle with center at the origin which is tangent to the line $2x + 3y = 39$

$2x + 3y = 39$

$2x + 3y = 0$

$r = \frac{39 - 0}{\sqrt{2^2 + 3^2}} = \frac{39}{\sqrt{13}} = 3\sqrt{13}$. ANS $x^2 + y^2 = 117$

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 2: FACTORING & APPLICATIONS

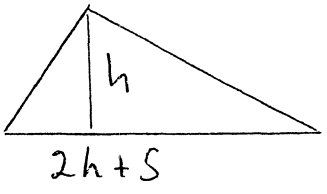
ANSWERS

A) 21

B) $-19, -17, -15$

C) $x(2x-3)(x-2)(x^2+2x+4)$

A) The base of a triangle is five more than twice the altitude to that base If the area of the triangle is 84, calculate the length of the base



$\frac{1}{2}h(2h+5) = 84$

$2h^2 + 5h - 168 = 0$

$(2h+21)(h-8) = 0$

$h = 8,\ 2h+5 = 21$

B) Find three consecutive odd integers such that the product of the first and the third added to the sum of all three is 234

$x,\ x+2,\ x+4$

$x(x+4) + (3x+6) = 234$

$x^2 + 7x - 228 = 0$

$(x+19)(x-12) = 0$

$x = -19$

ANS $-19, -17, -15$

C) Factor $2x^5 - 3x^4 - 16x^2 + 24x$

$x^4(2x-3) - 8x(2x-3) = x(2x-3)(x^3-8) =$

$x(2x-3)(x-2)(x^2+2x+4)$

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 3: TRIG EQUATIONS
NON-CALCULATOR

ANSWERS

A) $30^\circ, 150^\circ$

B) $2\pi/3, 4\pi/3$

C) $\pi/3, 2\pi/3, 4\pi/3, 5\pi/3$

A) Solve for $0^\circ \le \theta < 360^\circ$, $\sin\theta + \csc\theta = 2.5$

$\sin\theta = \frac{1}{2}$, $\csc\theta = 2$

$\theta = 30^\circ, 150^\circ$

B) Solve for $0 \le x < 2\pi$, $\dfrac{2\sin^2 x}{1-\cos x} = 1$

$2\sin^2 x = 1 - \cos x$

$2(1-\cos^2 x) = 1 - \cos x$

$2 - 2\cos^2 x = 1 - \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$, $\cos x = 1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0$, 0 is ext. Since $1-\cos x \neq 0$

C) Solve for $0 \le x < 2\pi$, $\tan 2x = -3\cot x$

$\dfrac{2\tan x}{1-\tan^2 x} = \dfrac{-3}{\tan x}$, $2\tan^2 x = -3 + 3\tan^2 x$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$ $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 4: QUADRATICS

ANSWERS

A) ± 8

B) 100

C) 4

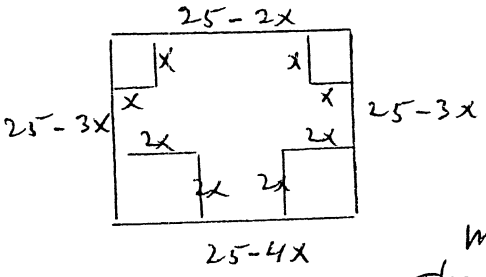
A) For what values of k will the equation $2x^2 - kx + 8 = 0$ have two equal real roots?

$k^2 - 4(2)(8) = 0$

$k^2 - 64 = 0$

$k = \pm 8$

B) The area of a square piece of tin is 625 sq in. Squares of equal size are cut out of the two top corners. Larger squares, each four times the area of a top corner square, are cut out of the two bottom corners. Calculate the perimeter of the resulting figure if its area is 535 sq. in.



$P = \left.\begin{matrix} 50 - 6x \\ 50 - 6x \\ 4x \\ 8x \end{matrix}\right\} = 100$

method 2: Solve To get $x = 3$.
Then add: $24 + 13 + 32 + 12 + 19 = 100$

C) If one root of $ax^2 + bx + c = 0$ is x = -2, b + c = 0, and a + b = 7; find the value of b

$x = -2,\ 4a - 2b + c = 0,\ c = -b,\ a = 7 - b$

$4(7 - b) - 2b - b = 0$

$28 - 7b = 0$

$b = 4$

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 5: SIMILAR POLYGONS

ANSWERS

A) 62.1

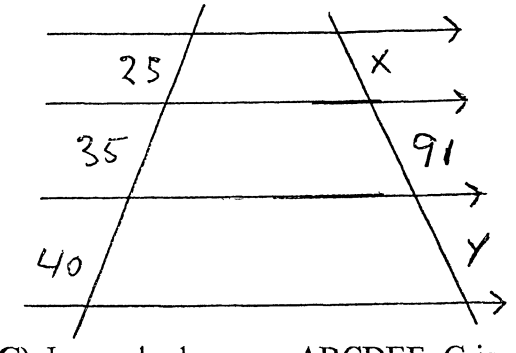
B) $x = 65,\ y = 104$

C) $(\sqrt{3} - 1) : 1$

A) There are two solid cubes made of the same material where the edge of one cube is three times the edge of the other. If the smaller cube weighs 2.3 grams, calculate to the nearest tenth, the weight of the larger cube.

$\frac{w}{2.3} = \frac{27}{1}$ $\quad w = 2.3(27) = 62.1$

B) In the figure shown, lines k, l, m, and n are parallel, with transversal segment lengths given. Calculate the sum of the lengths of segments x and y.

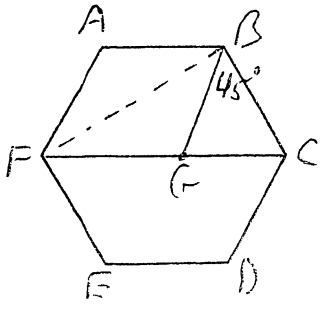


$\frac{91}{35} = \frac{13}{5}$

$x = \frac{25}{1} \cdot \frac{13}{5} = 65$

$y = \frac{40}{1} \cdot \frac{13}{5} = 104$

C) In regular hexagon ABCDEF, G is on $\overline{FC}$ so that $\angle CBG = 45^0$. Calculate in simple radical form, the ratio of $\overline{GC}$ *to* $\overline{CB}$.



$\angle FBC = 90^\circ$, so $\angle FBG = 45^\circ$, and BG is an $\angle$ bisector. Call $BC = 1$, Then $BF = \sqrt{3}$, and $FC = 2$. $FG = GC\sqrt{3}$ so $GC\sqrt{3} + GC = 2$, so $GC = \frac{2}{\sqrt{3}+1} = \sqrt{3} - 1$

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 6: ALGEBRA I ANYTHING

ANSWERS

A) 2

B) $(2x+1)/(x+3)$

C) 3, -5

A) Jeff and his wife Brenda can each weed the garden in four hours. One day after they had worked together weeding for one hour, their son Kyle helped them finish the work in one-half hour. How long would it have taken Kyle to weed the garden by himself?

$$\frac{3/2}{4} + \frac{3/2}{4} + \frac{1/2}{x} = 1 \qquad \frac{3}{2} + \frac{1}{x} = 2$$

$$\frac{3}{4} + \frac{3}{4} + \frac{1}{x} = 2 \qquad \frac{1}{x} = \frac{1}{2}, \; x = 2 \text{ hours}$$

B) Simplify: $\dfrac{15x^2 + 11x - 12}{25x^2 - 9} \div \dfrac{3x^2 + 13x + 12}{10x^2 + 11x + 3}$

$$\frac{(5x-3)(3x+4)}{(5x-3)(5x+3)} \cdot \frac{(5x+3)(2x+1)}{(3x+4)(x+3)} = \frac{2x+1}{x+3}$$

C) Solve for x: $|x-2| + |x+4| = 8$

$x - 2 + x + 4 = 8$ $\qquad$ $x - 2 + x + 4 = -8$

$2x + 2 = 8$ $\qquad$ $2x + 2 = -8$

$2x = 6$ $\qquad$ $2x = -10$

$x = 3$ $\qquad$ $x = -5$

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 7: TEAM QUESTIONS

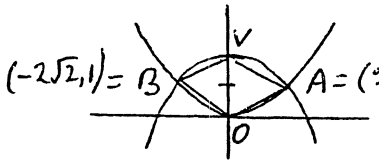
ANSWERS

A) $4\sqrt{2}$ D) ± 2

B) 6 E) $1+\sqrt{5}$

C) $45^\circ, 135^\circ, 225^\circ, 315^\circ$ F) 3

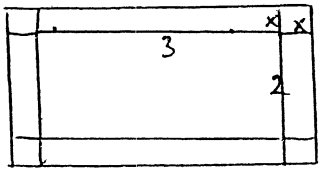
A) A parabola which has vertex V at the focus of the parabola $x^2 = 8y$, and focus at the origin O, intersects $x^2 = 8y$ at points A and B. Calculate the area of quadrilateral AVBO.



$x^2 = -8(y-2) = 8y$, $y = 1$, $x = \pm 2\sqrt{2}$. AVBO is rhombus,

Area $= \frac{1}{2}(4\sqrt{2})(2) = 4\sqrt{2}$

B) A two foot by three foot poster is framed with a border of uniform width. If the area of the border is the same as the area of the poster, calculate in <u>inches</u> the width of the border.



$2(x)(3+2x) + 2(2x) = 6$

$6x + 4x^2 + 4x - 6 = 0$

$4x^2 + 10x - 6 = 0$

$2x^2 + 5x - 3 = 0$

$(2x-1)(x+3) = 0$

$x = \frac{1}{2}$ ft $= 6$ inches

C) Solve for $0^0 \le \theta < 360^0$, $\cos 3\theta + \cos\theta = \sqrt{2}\cos 2\theta$

$2\cos 2\theta \cos\theta = \sqrt{2}\cos 2\theta$

$\cos 2\theta = 0$, $2\theta = 90^\circ, 270^\circ, 450^\circ, 630^\circ$

$\cos\theta = \sqrt{2}/2$ $\theta = 45^\circ, 315^\circ$

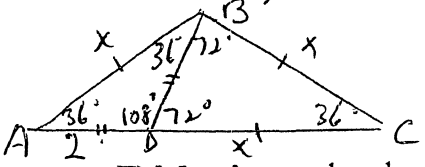
Ans $45^\circ, 135^\circ, 225^\circ, 315^\circ$

D) In the equation $ax^2 + bx + c = 0$, a, b, and c are relatively prime integers. If the product of its roots is $-\frac{8}{3}$, and the difference of its roots is $\frac{10}{3}$; calculate two possible values for b.

$rs = -\frac{8}{3}$ $\left(s + \frac{10}{3}\right)s = -\frac{8}{3}$, $3s^2 + 10s + 8 = 0$ $(3s+4)(s+2) = 0$

$r - s = \frac{10}{3}$ $s = -\frac{4}{3}$, $r = -\frac{4}{3} + \frac{10}{3} = 2$; $s = -2$, $r = -2 + \frac{10}{3} = \frac{4}{3}$. Eq is either $(3x+4)(x-2) =$ or $(3x-4)(x+2) =$

E) In ΔABC, AB = CB, $\angle B = 108^0$, D is on $\overline{AC}$ so that $\angle CBD$ *is twice* $\angle ABD$. If DA = 2, calculate CB.



$\frac{x}{2} = \frac{x+2}{x}$, $x^2 = 2x + 4$

$x^2 - 2x + 1 = 5$, $(x-1)^2 = 5$, $x = 1 + \sqrt{5}$

F) Mary's speed on her bike was 6 mph on the level, 4 mph downhill, and 12 mph uphill One day it took her one hour to go to Greg's house and return by the same route. How far in miles is it to Greg's house?

$\frac{d_1}{} + \frac{d_2}{} + \frac{d_3}{} + \frac{d_1}{} + \frac{d_2}{} + \frac{d_3}{} = 1$, $3d_1 + 2d_2 + d_3 + d_1 + 2d_2 + 3d_3 = 12$