

MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2004
ROUND 1 VOLUME & SURFACES

ANSWERS

A) _____

B) _____

C) _____

- A) The measure of the diagonal of a cube is $2\sqrt{6}$. Find the number of square units in the total surface area.
- B) A hollow metal sphere with diameter 0.20 m and thickness 0.01 m is melted and recast into small solid right cones with a base circumference of 0.20 m and height 0.03 m. If no volume is lost in this process, how many of the cones can be made (rounding down to the nearest whole number)?
- C) When a prism with a square base is enlarged by increasing its height by 12 but leaving the base unchanged, the surface area doubles while the volume triples. Find the original surface area.

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ROUND 2 PYTHAGOREAN RELATIONS

ANSWERS

A) _____

B) _____ meters

C) _____

- A) The diagonals of a rhombus have lengths of 16 and 30. Find the perimeter of the rhombus.
- B) A park has the shape of a right trapezoid ABCD with $\overline{AD} \perp \overline{AB}$ and $\overline{AD} \perp \overline{DC}$. AB=900 meters, AD=1200 meters, and BC=2000 meters. Two surveyors start at A and walk the perimeter at the same speed, one clockwise and the other counterclockwise. When they meet, how far are they from the nearest vertex?
- C) An altitude \overline{CD} is drawn to hypotenuse \overline{AB} of a right triangle with legs of 3 and 4. \overline{AD} , the shorter segment on the hypotenuse, is rotated 90° about the point D. The distance between B and the new location of A is $\frac{a}{b}\sqrt{c}$ where a, b, and c are integers, the radical is simplified, and a and b are relatively prime. Find a+b+c.

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ROUND 3 ALG 1: LINEAR EQUATIONS

ANSWERS

A) _____

B) _____

C) _____ ft/sec

- A) The average of three numbers is 20. The second number is four more than the first and the third is the sum of the first two. Find the square of the largest number.

B) Solve $\frac{45x + 43}{100} + \frac{x}{5} = 1 - \frac{5x + 6}{50}$

- C) Traveling from A to B with a headwind Shawn averaged 380 ft/sec; on the return from B to A with a tailwind Shawn averaged 420 ft/sec. Find Shawn's average speed for the entire trip.

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ROUND 4 ALG 1: FRACTIONS & MIXED NUMBERS

***** NO CALCULATORS ON THIS ROUND *****

ANSWERS

A) _____

B) _____

C) _____

A) Find the exact solution for $\frac{x-1}{2} - \frac{x-3}{4} - \frac{x-5}{6} = \frac{x-7}{8}$

- B) Consecutive unit fractions are those having numerators of 1 and denominators that are consecutive integers. Suppose a and b are consecutive unit fractions satisfying

$$a > \frac{\pi}{12} > b$$

Express $\frac{2ab}{a+b}$ as a simplified ratio of integers.

- C) At $A\frac{B}{C}$ minutes exactly after four o'clock p.m. (but before five o'clock) the minute and hour hands of a clock are perpendicular. Find the product of the two solutions for $A\frac{B}{C}$ and express the answer as a simplified mixed number.

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ROUND 5 INEQUALITIES & ABSOLUTE VALUE

ANSWERS

A) _____

B) _____

C) _____

A) Find all real x for which $(x^2 - 1)(x + 1) > (x^2 - 1)(x - 1)$

B) If the solution for $(b + ax - x^2)(14 - 2x) > 0$ is $x > 7$ or $-5 < x < 2$, find $a + b$

C) How many lattice points (points with two integer coordinates) are strictly inside the region bounded by

$$x = 0, y = 0, \text{ and } |2x + 2y - 9| = 7$$

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ROUND 6 ALG 1: EVALUATIONS

ANSWERS

A) _____

B) _____

C) _____

A) Consider the expression $2 * 120 / 5 * 3$ where $*$ means multiply and $/$ means divide. If A is the value of the expression using normal order of operations, and B is its value if division always takes precedence over multiplication, and C is its value if multiplication always takes precedence over division, evaluate $A - B + C$.

B) If $x \textcircled{\theta} y$ means x^y and $x \textcircled{\psi} y$ means $\sqrt[y]{x}$ find $[(4 \textcircled{\theta} 4) \textcircled{\psi} 2] \textcircled{\theta} 3$

C) If $\sqrt{x} + \sqrt{y} = 17$ and $x - y = 51$ find $x + y$