

MASSACHUSETTS MATHEMATICS LEAGUE
NOVEMBER 2004
ROUND 1 COMPLEX NUMBERS

ANSWERS

A) _____

B) _____

C) (_____, _____)

A) Find the exact value of xy if $(2 + i)^3 = 6x + 5yi$ with x and y real numbers

B) For a complex number z and its conjugate \bar{z} with $|z| = c$, $\frac{25}{z} + \frac{\bar{z}}{c^2} = 3 - 2i$. Express z in $a+bi$ form

C) Find all complex numbers x satisfying $x^2 = -2 + 8i\sqrt{3}$. Express answers in $a + bi$ form.

MASSACHUSETTS MATHEMATICS LEAGUE
NOVEMBER 2004
ROUND 2 ALGEBRA 1 ANYTHING

ANSWERS

A) _____

B) _____

C) _____

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- A) A total of \$8000 is invested in two mutual funds. At year's end one has paid 4%, the other 5% annual interest. If the combined interest after one year is \$350, how much was invested in the fund paying 4% interest?

Solve for x in terms of R: $R = \frac{1}{x} + \frac{1}{R}$

- C) Find the area of the triangle whose vertices are (1,2) (3,9) and (6,5)

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 ROUND 3 GEOMETRY: AREA

ANSWERS

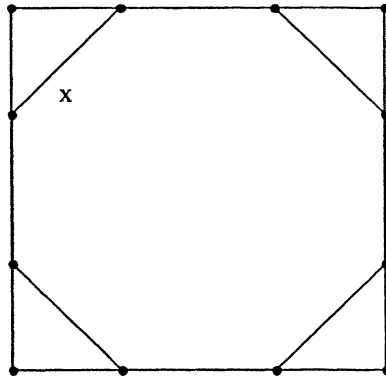
A) _____

B) _____

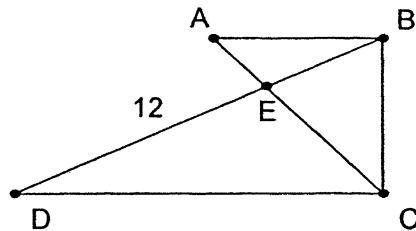
C) _____

- A) The largest square that fits in a 2 by 3 rectangle is one with area 178. Find the area of the 2 by 3 rectangle.

- B) If a regular octagon of side x is inscribed in a square of side 1 as shown below, find x in simplified radical form.



- C) Given right triangles ABC and BCD below with $\angle A=45^\circ$, $\angle D=30^\circ$, and $DE=12$, find the area of triangle BCE in simplest radical form.



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ROUND 4 FACTORING

ANSWERS

A) _____

B) _____

C) _____

A) Factor completely: $56x^2 - 78x - 108$

B) Factor completely $3xy - 20zw - 15xz + 4yw$

C) Factor completely: $a^2 + 6b^2 - 12c^2 - ac - bc - 5ab$

MASSACHUSETTS MATHEMATICS LEAGUE
NOVEMBER 2004
ROUND 5 TRIG: FUNCTIONS OF 30, 45, 60 & 90

***** NO CALCULATORS ON THIS ROUND *****

ANSWERS

A) _____

B) _____

C) _____

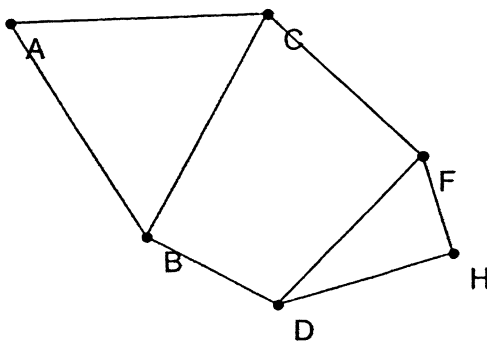
A) Find the exact value in simplified radical form of:

$$\csc(-675^\circ) + \cos(120^\circ) + 2\sin^2(420^\circ) - \tan^2(-330^\circ) + \cot^2(780^\circ)$$

B) Solve for all x , $0^\circ \leq x < 360^\circ$: $\sin(2x) - \sin(-x) = 0$

C) In the figure below, find the value of DH in simplified radical form if:

$$\sin(\angle FDH) = \cos(\angle A) = \cos(\angle ACB) = 0.5, \quad CF = FD, \quad AB = 10\sqrt{3}, \\ \cot(\angle CFD) = \cos(\angle CBD) = \cot(\angle H) = 0, \quad \text{and} \quad \cot(\angle BDH) = -1$$



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 ROUND 6 PLANE GEOMETRY: ANGLES

ANSWERS

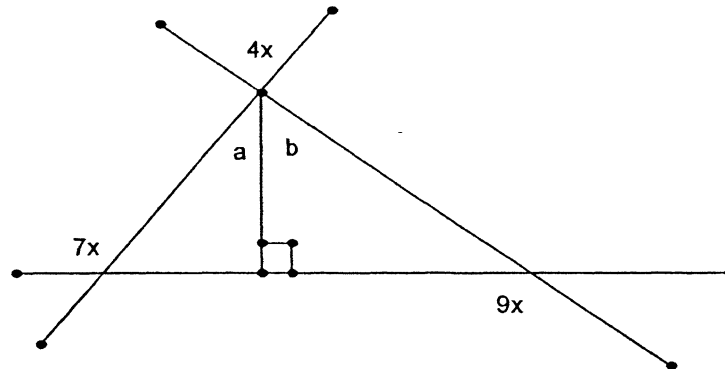
A) _____

B) _____ : _____

C) _____

- A) Given ABCD an isosceles trapezoid with diagonal \overline{AC} . If $m \angle B = 2x - 5$, $m \angle D = 3x$ and $m \angle ACD = x$, find $m \angle DAC$.

- B) Find the ratio of a to b expressed as a simplified ratio:



- C) The ratio of 2 angles' measures is 4:7 while the ratio of the complement of the smaller angle to the supplement of the larger angle is 7:16. Find the sum of the measures of the two angles.