

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 5 – FEBRUARY 2007  
ROUND 1 ALG 2: ALGEBRAIC FUNCTIONS**

**ANSWERS**

A) \_\_\_\_\_

B) \_\_\_\_\_

C) \_\_\_\_\_

A) Determine the domain of the function  $f(x) = \frac{1}{\sqrt{4x - 3x^2}}$ .

B) Given:  $f(0) = 0$  and  $f(x) = x - f(x - 1)$  for  $x > 0$   
Find  $f(2007)$ .

C) Find the largest 3-digit integer (in base 10) that can be formed from the distinct digits  $A$ ,  $m$  and  $b$ , given:

$$f(x) = Ax^2 \text{ and } g(x) = mx + b, \text{ where } A \neq 0 \text{ and } m \neq 0$$

$$f(g(x)) \equiv g(f(x)) \text{ ( i.e. an identity valid for all real values of } x \text{)}$$

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 5 – FEBRUARY 2007  
ROUND 2 ARITHMETIC / NUMBER THEORY**

**ANSWERS**

A) ( \_\_\_\_\_ , \_\_\_\_\_ )

B) \_\_\_\_\_

C) \_\_\_\_\_

A) Let  $A$  be the smallest positive integer value for which  $B = \frac{7A+1}{13}$  is also an integer.  
Find the ordered pair  $(A, B)$ .

B) The sequences of positive integers generated by  $7n + 2$  and  $11n + 4$  have exactly one two-digit integer in common. What is the largest three-digit integer that they have in common?

C) The product of the first 2007 positive prime numbers is divisible by several 3-digit positive integers of the form  $AAA_{(10)}$ . Find the sum of all 3-digit positive integers of this form.  
**Note:** 1 is not considered a prime number.

**MASSACHUSETTS MATHEMATICS LEAGUE**  
**CONTEST 5 – FEBRUARY 2007**  
**ROUND 3 TRIG: IDENTITIES AND/OR INVERSE FUNCTIONS**

**ANSWERS**

A) \_\_\_\_\_

B) \_\_\_\_\_

C) \_\_\_\_\_

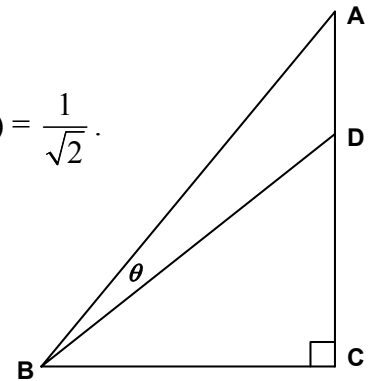
A) Given  $A = \sin^{-1}\left(\frac{35}{37}\right)$ ,  $B = \cos^{-1}\left(-\frac{15}{17}\right)$

Find  $\sin(A + B)$  as a simplified fraction.

B) In right  $\triangle ABC$ ,  $m\angle C = 90^\circ$ ,  $m\angle DBC = 45^\circ$ ,  $AB = 1 + \sqrt{2}$  and  $\tan(\theta) = \frac{1}{\sqrt{2}}$ .

Determine  $BC$  in simplified radical form.

Hint:  $BC = AB\cos(\angle ABC)$



C) Let  $\theta = \text{Arc cos}\left(\frac{1}{2x+1}\right)$ , where  $x > 0$ . Express the fraction  $\frac{x^2 + x}{2x + 1}$  as a single simplified fraction in terms of  $\cos(\theta)$ .

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 5 – FEBRUARY 2007  
ROUND 4 ALG 1: WORD PROBLEMS**

**ANSWERS**

A) \_\_\_\_\_

B) \_\_\_\_\_

C) \_\_\_\_\_

- A) Express Scripts uses three types of bottles to ship pills to their customers. Bottle  $A$  holds 12 fewer pills than bottle  $B$ . Bottle  $C$  holds twice as many pills as bottle  $A$ . If 2  $A$  bottles, 4  $B$  bottles and 3  $C$  bottles hold 240 pills. How many pills does an  $A$  bottle hold?
- B) The first book in a trilogy by Anne McCaffrey contains 67 more pages of text than the second, while the third contains 24 fewer pages of text than the second. The total number of pages of text in the trilogy is 751 pages. If all the chapters in the first book contain exactly  $P$  pages of text, where  $P > 1$ , what is the maximum number of chapters possible in the first book?
- C) Of the 585 girls at Northeast Statistics High School last year, 170 played fall sports, 165 played winter sports and 150 played spring sports. Twenty two girls played sports in all three seasons, while 80 played only in the fall, 84 played only in the winter, and 71 played only in the spring. How many girls did not play sports in any of the three seasons?

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 5 – FEBRUARY 2007  
ROUND 5 GEOMETRY: CIRCLES**

**ANSWERS**

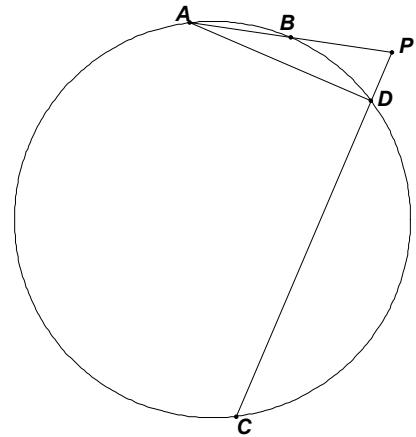
- A) \_\_\_\_\_ °  
 B) \_\_\_\_\_  
 C) \_\_\_\_\_

**\*\*\*\*\* NO CALCULATORS ON THIS ROUND \*\*\*\*\***

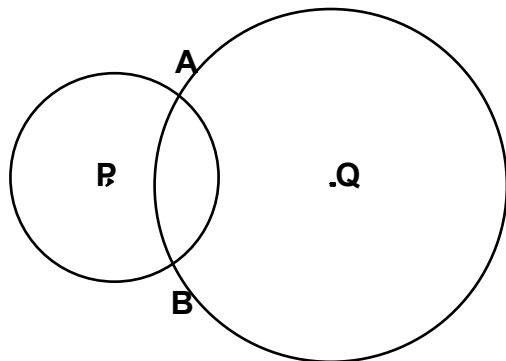
**If necessary, all required lengths must be expressed in terms of simplified radicals.**

- A) Quadrilateral  $ABCD$  is inscribed in a circle.  $m\angle D = 75^\circ$ ,  $\widehat{AB} = x^2$ ,  $\widehat{BC} = 5x$  and  $\widehat{CD} = 6x$   
 Find  $m\angle A$ .

- B) Given:  $AB = 3$ ,  $PB = 4$ ,  $CD = 12$  and  $m\angle ADP = 90^\circ$   
 Determine  $DA$ .



- C) Circle  $P$  and circle  $Q$  have radii 5 and 7 respectively and  $PQ = 8$ . Find the exact length of the common chord  $\overline{AB}$ .



**MASSACHUSETTS MATHEMATICS LEAGUE**  
**CONTEST 5 – FEBRUARY 2007**  
**ROUND 6 ALG 2: SEQUENCES AND SERIES**

**ANSWERS**

A) \_\_\_\_\_

B) \_\_\_\_\_

C) \_\_\_\_\_

A) The symbol  $\Sigma$  in mathematics represents a summation, the addition of several terms of a specific type. For example,  $\sum_{k=1}^{k=4} k^2 = 1^2 + 2^2 + 3^2 + 4^2 = 30$ .

Evaluate:  $\sum_{k=2}^{k=6} (3k - 2)$

B) Let  $t_1 = (2^{-1} + 2^{-2} + 2^{-3} + \dots)$ ,  $t_2 = (3^{-1} + 3^{-2} + 3^{-3} + \dots)$ , ...  $t_n = ((n+1)^{-1} + (n+1)^{-2} + (n+1)^{-3} + \dots)$   
Determine the minimum number of terms that must be added so that the sum exceeds 2.

C) The harmonic mean of nonzero numbers  $a$  and  $b$  is defined as  $\frac{2ab}{a+b}$ .

Given a sequence 1,  $x$ ,  $y$ , 2 such that  $x$  is the harmonic mean between 1 and  $y$  and  $y$  is the harmonic mean between  $x$  and 2. What is the sum of  $x$  and  $y$ ?

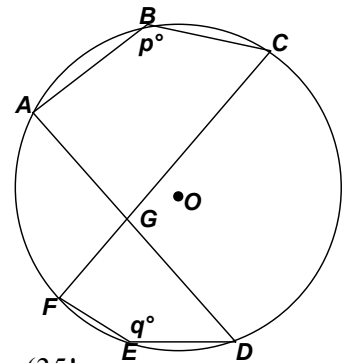
**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 5 – FEBRUARY 2007  
ROUND 7 TEAM QUESTIONS**

**ANSWERS**

- A) \_\_\_\_\_ D) \_\_\_\_\_ feet  
 B) \_\_\_\_\_ E) \_\_\_\_\_ °  
 C) \_\_\_\_\_ F) \_\_\_\_\_

- A) Given  $f(x) = 3x - 1$  and  $g(x) = 5x + 2$   
 Determine all possible  $x$  for which  $g^{-1}(f(x)) = f^{-1}(g(x))$
- B) Let  $P$  = the product of the natural numbers from 1 to 25 inclusive.  
 Let  $S$  = the sum of the natural numbers from 1 to 25 inclusive.  
 Let  $N = P + S$   
 What is the sum of the rightmost 7 digits of  $N$ ?
- C) In  $\triangle ABC$ ,  $m\angle A = 30^\circ$ ,  $BC = 7$  and it is known that  $AB$  is an integer.  
 For how many different integer values is  $\triangle ABC$  an acute triangle?
- D) Building #1 is twice as tall as building #2. Each building has a lobby with 12 foot ceilings and additional floors with ceilings of uniform height. However, the ceiling heights in building #1 are 6 inches more than in building #2 and building #1 has 3 more floors than building #2. If the ceiling heights in both buildings must be at least 8 feet high, what is the minimum height (in feet) of building #1?

- E) Given:  $m\angle ABC = p^\circ$  and  $m\angle FED = q^\circ$   
 $A, B, C, D, E$  and  $F$  are points on circle  $O$  and  
 $G$  is the point of intersection of chords  $\overline{AD}$  and  $\overline{CF}$   
 as indicated in the diagram  
 Determine the degree measure of  $\angle AGC$  in terms of  $p$  and  $q$ .



- F) Given the sequence  $t_1 = (29! + 30! + 31!)$ ,  $t_2 = (30! + 31! + 32!)$ , ...,  $t_7 = (35! + 36! + 37!)$   
 Let  $p_k$  be the largest prime factor of  $t_k$ .  
 Determine the value of  $\sum_{k=1}^{k=7} p_k$ .

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 5 – FEBRUARY 2007 ANSWERS**

**Round 1 Alg 2: Algebraic Functions**

- A)  $0 < x < 4/3$                       B) 1004                      C) 910

**Round 2 Arithmetic/ Number Theory**

- A) (11, 6)                      B) 961                      C) 1665

**Round 3 Trig Identities and/or Inverse Functions**

- A)  $-\frac{429}{629}$                       B)  $\frac{\sqrt{6}}{6}$                       C)  $\frac{1 - \cos^2 \theta}{4 \cos \theta}$

**Round 4 Alg 1: Word Problems**

- A) 16                      B) 101                      C) 236

**Round 5 Geometry: Circles**

- A) 55                      B)  $3\sqrt{5}$                       C)  $5\sqrt{3}$

**Round 6 Alg 2: Sequences and Series**

- A) 50                      B) 4                      C) 2.7 (or equivalent)

**Team Round**

- A)  $-3/2$                       B) 14                      C) 6

- D) 62                      E)  $(360 - p - q)^\circ$

- F) 221



**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 5 – FEBRUARY 2007 SOLUTION KEY**

**Round 1**

- A) The radicand must be positive to insure that the denominator is real and nonzero.

$$4x - 3x^2 = x(4 - 3x) > 0 \rightarrow x > 0 \text{ and } x < 4/3 \rightarrow \underline{\mathbf{0 < x < 4/3}}$$

B)  $f(1) = 1 - f(0) = 1, \quad f(2) = 2 - f(1) = 2 - 1 = 1$

$$f(3) = 3 - f(2) = 3 - 1 = 2, \quad f(4) = 4 - f(3) = 4 - 2 = 2$$

$$f(5) = 5 - f(4) = 5 - 2 = 3, \quad f(6) = 6 - f(5) = 6 - 3 = 3$$

In general, for even  $n$ ,  $f(n) = n/2$ ; for odd  $n$ ,  $f(n) = (n + 1)/2 \rightarrow f(2007) = \underline{\mathbf{1004}}$

C)  $A(mx + b)^2 \equiv m(Ax^2) + b \rightarrow Am^2x^2 + 2Ambx + Ab^2 \equiv Amx^2 + b$

**Note:**  $\equiv$  denotes this is an identity, not just an equation. It is true for all values of  $x$ .

Equating coefficients and recalling that  $A \neq 0, m \neq 0$ ,

$$Am^2 = Am \rightarrow Am(1 - m) = 0 \rightarrow m = 1$$

$$2Amb = 0 \rightarrow b = 0$$

$$Ab^2 = b \rightarrow A \text{ can be any nonzero digit.}$$

Choosing  $A = 9$ , maximizes the 3-digit number: **910**

**Round 2**

- A) Substituting  $A = 1, 2, 3, \dots$  produces the sequence 8, 15, 22, ....

The first multiple of 13 in this sequence occurs when  $A = 11, B = 78/13 = 6$

Thus,  $(A, B) = \underline{\mathbf{(11, 6)}}$ .

- B) The expressions  $7n + 2$  and  $11n + 4$  generate the sequences

2, 9, 16, 23, 30, 37, ... and 4, 15, 26, 37, ...

Clearly, the two-digit integer they have in common is 37.

The next common integer can be found by adding 77, the least common multiple of 7 and 11.

To find the largest three-digit integer  $A$  that they have in common, solve the inequality

$$A = 37 + 77k < 1000 \text{ over the integers. } k < 963/77 = 12^+ \rightarrow k = 12 \rightarrow A = 37 + 924 = \underline{\mathbf{961}}$$

- C) **111** = 3(37) – ok

$$\mathbf{222} = 2(3)(37) - \text{ok}$$

$$333 = 3(111) = 3^2(37) - \text{fails because of the repeated prime.}$$

$$444 = 4(111) = 2^2(3)(37) \text{ fails}$$

$$\mathbf{555} = 3(5)(37) - \text{ok}$$

$$666 = (2)3^2(37) - \text{fails}$$

$$\mathbf{777} = 3(7)(37) - \text{ok}$$

$$888 = 2^3(3)(37) - \text{fails}$$

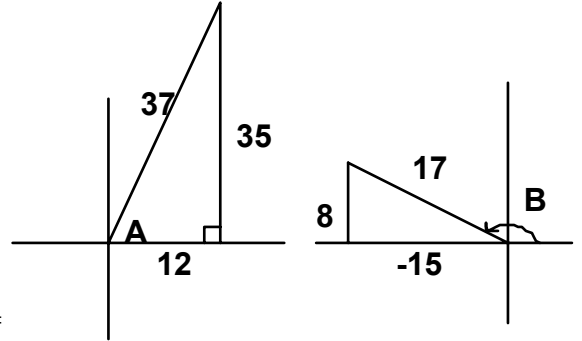
$$999 = 3^3(37) - \text{fails}$$

Thus, the required sum is  $(1 + 2 + 5 + 7)(111) = (15)(111) = \underline{\mathbf{1665}}$ .

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 5 – FEBRUARY 2007 SOLUTION KEY**

**Round 3**

$$\begin{aligned} \text{A) } \sin(A + B) &= \sin A \cos B + \cos A \sin B = \frac{35}{37} \cdot \frac{-15}{17} + \frac{12}{37} \cdot \frac{8}{17} \\ &= \frac{-525 + 96}{629} = \boxed{\frac{429}{629}} \end{aligned}$$

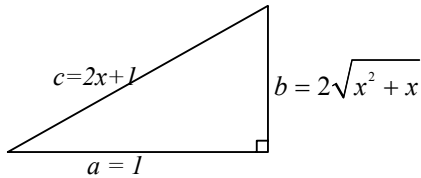


$$\begin{aligned} \text{B) } \tan(\theta) &= \frac{1}{\sqrt{2}} \text{ for an acute angle } \theta \rightarrow (\cos(\theta), \sin(\theta)) = \\ & \left( \frac{\sqrt{2}}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) \end{aligned}$$

$$\cos(\theta + 45^\circ) = \cos(\theta)\cos(45^\circ) - \sin(\theta)\sin(45^\circ) = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{6}}$$

$$BC = AB \cos(\theta + 45^\circ) = \frac{(\sqrt{2}+1)(\sqrt{2}-1)}{\sqrt{6}} = \frac{1}{\sqrt{6}} = \boxed{\frac{\sqrt{6}}{6}}$$

C)



$$\text{Then } \frac{x^2+x}{2x+1} = \frac{b^2/4}{c} = \frac{1}{4} \cdot \frac{b}{1} \cdot \frac{b}{c} = \frac{1}{4} \tan \theta \sin \theta = \frac{1}{4} \frac{\sin^2 \theta}{\cos \theta} = \boxed{\frac{1 - \cos^2 \theta}{4 \cos \theta}}$$

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 5 – FEBRUARY 2007 SOLUTION KEY**

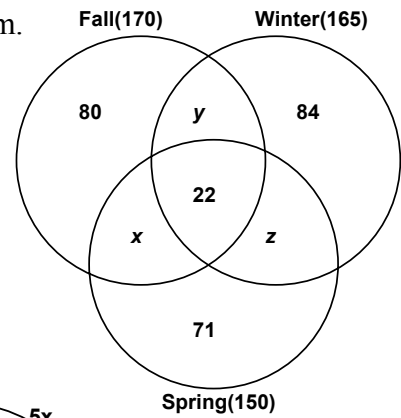
**Round 4**

- A) Let  $B$  hold  $x$  pills. Then  $A$  holds  $x - 12$  and  $C$  holds  $2(x - 12)$   
 $2A + 4B + 3C = 240 \rightarrow 2x - 24 + 4x + 6x - 72 = 240 \rightarrow 12x = 336 \rightarrow x = 28 \rightarrow A = \underline{16}$
- B) Let  $x = \#$  pages of text in book 2. Then  $1^{\text{st}} = x + 67$  and  $3^{\text{rd}} = x - 24$   
 $3x + 43 = 751 \rightarrow x = 236 \rightarrow 1^{\text{st}}$  book has 303 pages.  $303 = 3(101)$   
 Since both these factors are prime, the first book either has 3 chapters of 101 pages each or 101 chapters of 3 pages each. Thus, the maximum number of chapters is 101.

- C) The given information translates into the following Venn Diagram.

Fall  $\rightarrow x + y = 68$   
 Winter  $\rightarrow y + z = 59$   
 Spring  $\rightarrow x + z = 57$   
 Fall-Winter  $\rightarrow x - z = 9$

Thus,  $x = 33$ ,  $y = 35$  and  $z = 24$  and the total number of girls participating in at least one sport is  $170 + 84 + 71 + z = 349$   
 $\rightarrow 585 - 349 = \underline{236}$  non-participants



**Round 5**

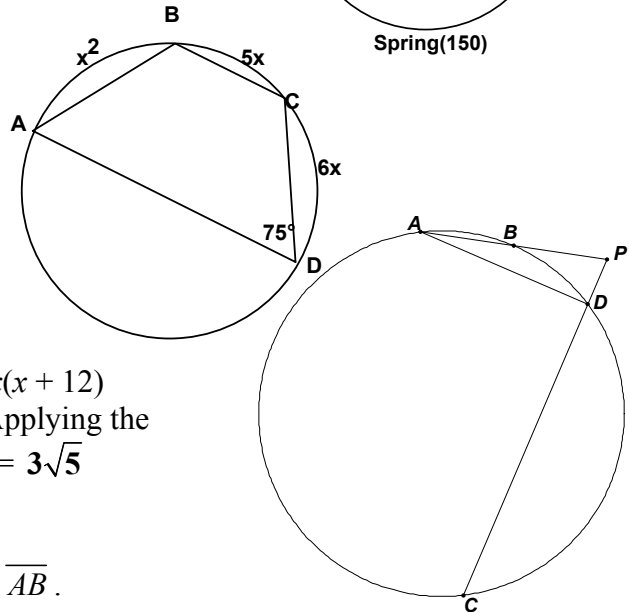
- A) Appealing to the diagram, as inscribed angles,

$$m\angle D = \frac{1}{2}(x^2 + 5x) = 75$$

$$\rightarrow x^2 + 5x - 150 = (x + 15)(x - 10) = 0$$

$$\rightarrow x = 10$$

$$\rightarrow m\angle A = \frac{1}{2}(5x + 6x) = \frac{1}{2}(110) = \underline{55^\circ}$$



- B) Let  $PD = x$ . Then  $PB(PA) = PD(PC) \rightarrow 4(7) = x(x + 12)$   
 $\rightarrow x^2 + 12x - 28 = (x + 14)(x - 2) = 0 \rightarrow x = 2$  Applying the  
 Pythagorean theorem,  $DA^2 = 49 - 4 = 45 \rightarrow DA = \underline{3\sqrt{5}}$

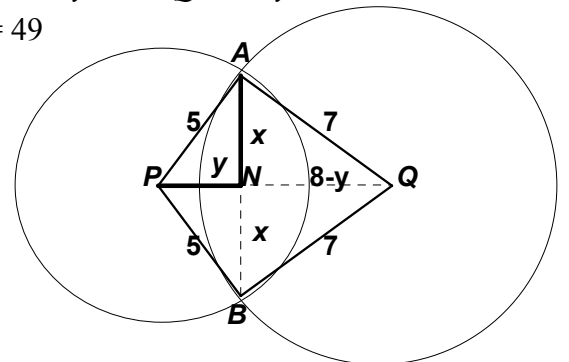
- C)  $APBQ$  is a kite and  $\overline{PQ}$  perpendicularly bisects  $\overline{AB}$ .

Let  $N$  be the point of intersection of  $\overline{PQ}$  and  $\overline{AB}$ . Let  $AN = x$ ,  $PN = y$  and  $NQ = 8 - y$

Then:  $x^2 + y^2 = 25$  and  $(8 - y)^2 + x^2 = 49 \leftrightarrow x^2 + y^2 - 16y + 64 = 49$

Substituting,  $25 - 16y + 64 = 49 \rightarrow y = \frac{5}{2} \rightarrow x^2 = \frac{75}{4}$

$\rightarrow x = \frac{5\sqrt{3}}{2} \rightarrow AB = \boxed{5\sqrt{3}}$



**MASSACHUSETTS MATHEMATICS LEAGUE  
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**Round 6**

A) For  $k = 2$  to  $6$ , the expression  $3k - 2$  produces the numbers  $4, 7, 10, 13$  and  $16$ .  
The sum of these  $5$  numbers is **50**.

B) Each of these terms is the sum of an infinite geometric sequence. Applying  $\frac{a}{1-r}$ ,

$$t_1 = 1, t_2 = 1/2, t_3 = 1/3, t_4 = 1/4, \text{ etc}$$
$$1 + 1/2 + 1/3 + 1/4 = 25/12 > 2 \rightarrow n = \underline{4}$$

C) By definition,  $x = \frac{2y}{1+y}$  and  $y = \frac{4x}{x+2}$ . Substituting for  $x$  in the second equation,

$$y = \frac{4\left(\frac{2y}{1+y}\right)}{\frac{2y}{1+y} + 2} = \frac{8y}{4y+2} = \frac{4y}{2y+1} \rightarrow 2y^2 + y = 4y \rightarrow 2y^2 - 3y = y(2y - 3) = 0 \rightarrow y = \frac{3}{2}$$

$$\text{and } x = \frac{3}{5/2} = \frac{6}{5}. \text{ Adding the required sum is } \frac{3}{2} + \frac{6}{5} = \frac{5+12}{10} = \underline{2.7}$$

**MASSACHUSETTS MATHEMATICS LEAGUE  
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**Team Round**

A)  $f(x) = 3x - 1$  and  $g(x) = 5x + 2 \rightarrow f^{-1}(x) = (x + 1)/3$  and  $g^{-1}(x) = (x - 2)/5$

$$g^{-1}(f(x)) = g^{-1}(3x - 1) = \frac{3x - 1 - 2}{5} = \frac{3x - 3}{5}$$

$$f^{-1}(g(x)) = f^{-1}(5x + 2) = \frac{(5x + 2) + 1}{3} = \frac{5x + 3}{3}$$

Equating and cross multiplying  $\rightarrow 9x - 9 = 25x + 15 \rightarrow 16x = -24 \rightarrow x = \underline{-3/2}$ .

B)  $S = 25(26)/2 = 325$

$$\left[ \frac{25}{5} \right] + \left[ \frac{25}{25} \right] = 6 \rightarrow \text{the rightmost 6 digits of } 25! \text{ are zeros.}$$

Note:  $[x]$  denotes the greatest integer in  $x$ , i.e. the largest integer  $\leq x$

(5, 10, 15 and 20 each contain 1 factor of 5 and 25 contains 2 factors of 5)

Thus, the rightmost 7 digits of  $N$  are  $x000325$ , where we need only determine  $x$ .

Since there are no more factors of 5 in  $25!$  and an excess of 2s, we know that  $x$  must be nonzero and even. Which is it? 2, 4, 6 or 8?

The prime factorization of  $25!$  is  $2^{x_1} 3^{x_2} 5^{x_3} 7^{x_4} 11^{x_5} 13^{x_6} 17^{x_7} 19^{x_8} 23^{x_9}$

We have already determined that  $x_3 = 6$  and the following are easily verified:

$$x_6 \dots x_9 = 1, x_5 = 2, x_4 = 3, x_2 = 10 \text{ and } x_1 = 16$$

$$x_5 = \left[ \frac{25}{11} \right] + \left[ \frac{25}{121} \right] = 2 \qquad x_4 = \left[ \frac{25}{7} \right] + \left[ \frac{25}{49} \right] = 3$$

$$x_2 = \left[ \frac{25}{3} \right] + \left[ \frac{25}{9} \right] + \left[ \frac{25}{27} \right] = 8 + 2 = 10$$

$$x_1 = \left[ \frac{25}{2} \right] + \left[ \frac{25}{4} \right] + \left[ \frac{25}{8} \right] + \left[ \frac{25}{16} \right] + \left[ \frac{25}{32} \right] = 12 + 6 + 3 + 1 = 22$$

Thus, the prime factorization of  $25! = 2^{22} 3^{10} 5^6 7^3 11^2 13^1 17^1 19^1 23^1$ .

Pulling out the factors of 10,  $25! = 10^6 (2^{16} 3^{10} 7^3 11^2 13^1 17^1 19^1 23^1)$

The rightmost digit of the product in parentheses is the digit  $x$  we need.

$$2^{16} = (2^4)^4 = (6)^4 \text{ ends in } 6$$

$$3^{10} = (3^4)^2 3^2 = (1)^2 3^2 \text{ ends in } 9$$

$$7^3 \text{ ends in } 3$$

It's left to you to verify that  $11^2 13^1 17^1 19^1 23^1$  ends in 7

The product  $(6)(9)(3)(7)$  ends in 4.

Thus, the last 7 digits are 4000325 and the sum is **14**.

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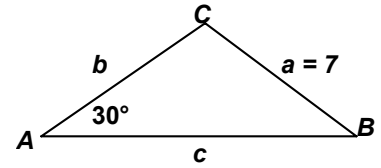
**Team Round - continued**

- C) Since the given information represents two sides and a **non**-included angle,  $\triangle ABC$  might not exist or there could be exactly 1 or 2 non-congruent triangles that satisfy the stated conditions.

Let  $(BC, AB) = (a, c)$ . Then using the law of Sine,  $\frac{\sin 30^\circ}{a} = \frac{\sin C}{c} \rightarrow \sin C = \frac{c}{2a}$

Case 1:  $\frac{c}{2a} > 1 \rightarrow$  no solution ( $\sin C$  can not exceed 1)

Case 2:  $\frac{c}{2a} = 1 \rightarrow \triangle ABC$  is a right triangle (in fact 30-60-90)



Case 3:  $\frac{1}{2} < \frac{c}{2a} < 1 \rightarrow a < c < 2a \rightarrow$  2 solutions 1 acute and 1 obtuse

(Ex:  $C = 45^\circ$  and  $135^\circ$  or any pair of supplementary angles  $\theta$  and  $180 - \theta$ , where  $\theta > 30^\circ$ )

Case 4:  $\frac{c}{2a} = \frac{1}{2} \rightarrow c = a \rightarrow m\angle A = m\angle C = 30^\circ \rightarrow m\angle B = 120^\circ \rightarrow \triangle ABC$  is obtuse

Case 5:  $0 < \frac{c}{2a} < \frac{1}{2} \rightarrow c < a$ ,  $m\angle C < 30^\circ$  or  $m\angle C > 150^\circ$ . The latter is impossible, but if  $m\angle C < 30^\circ$ , then  $m\angle B > 120^\circ$  and  $\triangle ABC$  is obtuse.

Thus, all the acute triangles arise from case 3. We have  $7 < c < 14 \rightarrow 8 \leq c \leq 13$ .

The number of integer values of  $c$  is  $13 - 8 + 1 = \underline{6}$

Note, in general, the solution is  $(2a - 1) - (a + 1) + 1 = a - 1$

- D) Let building #1 have  $(N + 3)$  floors w/ ceilings  $H$  feet high and building #2 have  $N$  floors with ceilings  $(H - 0.5)$  feet high.

$$12 + (N + 3)H = 2(12 + N(H - 0.5)) \rightarrow 12 + NH + 3H = 24 + 2NH - N$$

$$3H - NH = 12 - N \rightarrow H = \frac{12 - N}{3 - N}$$

$N = 1 \rightarrow H = 11/2$  (rejected – ceilings not at least 8 feet high)

$N = 2 \rightarrow H = 10/1$

Thus, building #1 is  $12 + 5(10) = \underline{62}$  feet tall.

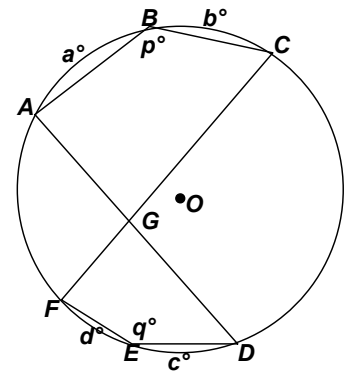
E) As an angle formed by two intersecting chords,

$$m\angle AGC = \frac{1}{2}(a + b + c + d)$$

As inscribed angles,  $p = \frac{1}{2}(360 - (a + b))$  and  $q = \frac{1}{2}(360 - (c + d))$

Thus,  $p + q = 360 - \frac{1}{2}(a + b + c + d) = 360 - m\angle AGC$

$$\rightarrow m\angle AGC = \underline{\underline{(360 - p - q)^\circ}}$$



F) Factoring the expression  $n! + (n + 1)! + (n + 2)!$ , we have  $n!(1 + (n + 1) + (n + 1)(n + 2))$

$$= n!(n + 2) + (n + 1)(n + 2) = n!(n + 2)(1 + (n + 1)) = n!(n + 2)^2$$

Thus, for each triple, we take either the largest prime  $\leq n$  or  $(n + 2)$ , if  $(n + 2)$  is prime.

$$(t_1, t_2, \dots, t_7) = (29!(31)^2, 30!(32)^2, 31!(33)^2, 32!(34)^2, 33!(35)^2, 34!(36)^2, 35!(37)^2)$$

$$\rightarrow (p_1, p_2, \dots, p_7) = (31, 29, 31, 31, 31, 31, 37) \rightarrow \underline{\underline{221}}$$

**Addendum:**

In round 6, question B, the original wording was problematic:

Let  $t_1 = (2^{-1} + 2^{-2} + 2^{-3} + \dots)$ ,  $t_2 = (3^{-1} + 3^{-2} + 3^{-3} + \dots)$ , ...  $t_n = ((n+1)^{-1} + (n+1)^{-2} + (n+1)^{-3} + \dots)$   
How many terms of this sequence must be added before the sum exceeds 2?

Is the question asking for the maximum number of terms which could be added without exceeding 2 or the minimum number of terms for which the sum exceeded 2?

Answers of 3 or 4 were accepted.

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In round 6, question C, the original wording of the problem omitted the phrase “of nonzero numbers”.

Unfortunately, in this case, the sequences 1, 3/2, 6/5, 2 and 1, 0, 0, 2 both satisfy the stated requirements.

Answers of 2.7 or 0 or both were accepted.