MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008
ROUND 1 ALG 2: ALGEBRAIC FUNCTIONS

ANSWERS

A) ______________________________

B) { ___________________________ }

C) ___________________________ feet

A) Given f: $f(x) = \dfrac{x}{x+1}$, find $f(f(x))$ in simplified form.

B) Definition: $g(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \end{cases}$

If f: $f(x) = x\left(\dfrac{1+g((x+2)(3-x))}{2}\right)$ and the domain of f is $\{x \mid x \le -2 \text{ or } x \ge 3\}$, determine the range of f.

C) Initially, a collapsing rectangle R_1 has a length of 40 feet and a width of 30 feet. Its length decreases at a constant rate of 4 ft/sec, while its width decreases at a constant rate of 3 ft/sec. At the same time, an expanding rectangle R_2 has a length of 4 feet and a width of 3 feet. Its length increases at a constant rate of 2 ft/sec and its width increases at a constant rate of 6 ft/sec. Determine the <u>sum</u> of the perimeters of the two rectangles at the time when the area of R_1 is 108 square feet greater than the area of R_2.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008
ROUND 2 ARITHMETIC / NUMBER THEORY

ANSWERS

A) ______________________________

B) ______________________________

C) ______________________________

A) What is the units digit of the product $7^{218}(3^{507})$?

B) Let $x^2 - y^2 = 31$, where x and y are positive integers.
If $N = xy$, how many positive factors does N have?

C) Using only the prime digits 2, 3, 5 and 7 without repetition, form all possible positive integers in base 10 with at most 4 digits (i.e. less than 10,000), that will be divisible by 15.
What is the sum of all these positive integers?

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008
ROUND 3 TRIG: IDENTITIES AND/OR INVERSE FUNCTIONS

ANSWERS

A) _______________________________

B) _______________________________

C) _______________________________

A) Solve for θ, where $0° \leq \theta < 360°$, if $\csc(2\theta) + \cot(2\theta) = 1$

B) Given: $\cos(40°) = k$ and $\sin(x) = 1 - 2k^2$
What are the possible values of x between 0° and 360° exclusive?

C) Determine the positive integer n for which

$$\sin\left(Arc\cos\left(-\frac{n}{11}\right) + Arc\tan\left(-\frac{1}{2\sqrt{6}}\right)\right) = \frac{53}{55}$$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008
ROUND 4 ALG 1: WORD PROBLEMS

ANSWERS

A) ______________________________

B) ____________________________ %

C) ____________________________ oz.

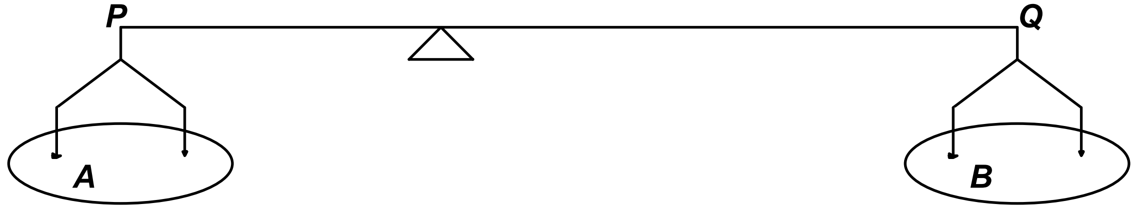
A) 35 students registered for enrichment courses in a summer program at Brewster Academy. Here is registration information for the science courses.

Course	# students registered
Biology	19
Chemistry	17
Physics	11
Chemistry and Physics	5
Biology and Physics	7
Biology and Chemistry	12
All 3 courses	2

How many of the 35 students did not register for any of these science courses?

B) Three men work at equal rates work 8 hours per day. If they complete 80% of a job in four 8-hour days, what percent of the job would be completed by one of these men in 6 hours?

C) A supply of cylinders of equal but unknown mass are available and used as follows:
8 cylinders in pan *A* balance a single 9 oz. cube in pan *B*.
A single 25 oz cube in pan *A* balances 2 cylinders in pan *B*.
How much (in ounces) does each cylinder weigh?
Note that the fulcrum is not at the midpoint of $\overline{PQ}$.



MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008
ROUND 5 GEOMETRY: CIRCLES

ANSWERS

A) ________________________ ft/min

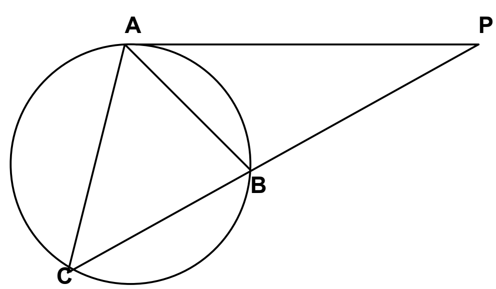
B) ____________________________

C) ____________________________

******* NO CALCULATORS ON THIS ROUND *******

A) A wheel of radius 6 inches rotates at 2 revolutions per second. In terms of π, how fast does a point on the circumference turn in feet per minute?

B) $\overline{PA}$ is tangent to circle O at A, $PA = 5x - 3$, $PB = 3x - 1$, $BC = 7x - 11$ and $AC = 2x + 3$ Compute the perimeter of ΔAPC.



C) In circle O, perpendicular chords $\overline{AB}$ and $\overline{CD}$ intersect at point P. $AP = 12$, $PB = 28$ and $CP = 14$. Compute $AD - PO$.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008
ROUND 6 ALG 2: SEQUENCES AND SERIES

ANSWERS

A) ______________________________

B) ______________________________

C) ______________________________

A) 3^2, 5^2, 7^2 are the 2^{nd}, 6^{th} and 12^{th} terms in an arithmetic sequence. What is the 14^{th} term?

B) Find the first term in the arithmetic sequence -2, 5, 12, 19, … that is larger than the 10^{th} term in the geometric sequence -0.75, 1.5, -3, 6, …

C) Given: $\begin{cases} A_{N+2} = 2A_{N+1} + 3A_N \quad \text{for } N \ge 1 \\ A_2 = 4 \\ A_5 = 17 \end{cases}$ <u>Compute</u> $A_1 + A_6$.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008
ROUND 7 TEAM QUESTIONS

ANSWERS

A) ____________________________ D) ____________________________

B) ____________________________ E) ____________________________

C) ____________________________ F) ____________________________

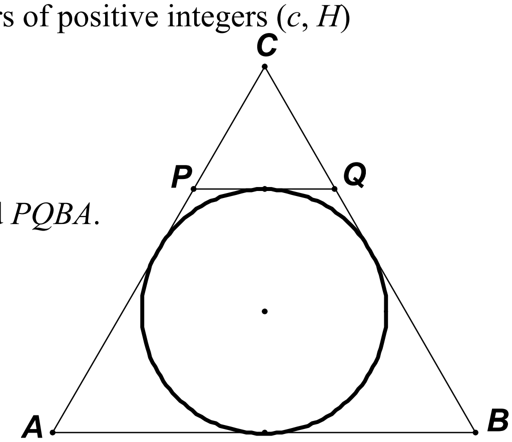
A) Given: $\begin{cases} f(x) = 8x \\ f(g(x)) = 27x^3 \\ f(g(h(x))) = 27x^{15} \end{cases}$ Compute: $h^{-1}(1024) \cdot g^{-1}(-1)$

B) What is the smallest value of N for which N! ends in exactly 2008 zeros?

C) Given: $Cos^{-1}(2x) - Sin^{-1}(x) = 5\pi / 6$
Compute $x < 0$.

D) In still water, a motorboat travels at 18 mph.
A fisherman leaves a dock and travels downstream on a river with a constant current of c mph for an hour, turns the motor off and fishes for half an hour, floating with the current. He then immediately starts the motor and travels back upstream, returning to the dock. The upstream trip takes H hours. Determine all possible ordered pairs of positive integers (c, H) satisfying these conditions.

E) Given: $\overline{PQ} \parallel \overline{AB}$, $PQ : AB = 1 : 3$ and circle O is inscribed in trapezoid $PQBA$
Compute the ratio of the area of circle O to the area of trapezoid $PQBA$.



F) The arithmetic average of n numbers is M. If 1 is added to the first number, 2 to the second number and so on until n is added to the n^{th} number, then the average of the new numbers is A. If $M : A = 2 : 3$ and $n > 1,000,000$ then determine the minimum value of M.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008 SOLUTION KEY

Round 1

A) $f\left(\dfrac{x}{x+1}\right)=\dfrac{\dfrac{x}{x+1}}{\dfrac{x}{x+1}+1}=\dfrac{x}{x+(x+1)}=\underline{\boldsymbol{\dfrac{x}{2x+1}}}$

B) The expression $(x+2)(3-x)$ is:

a) positive for $-2 < x < 3$
b) negative for $x < -2$ or $x > 3$
c) zero for $x = -2$ or $+3$

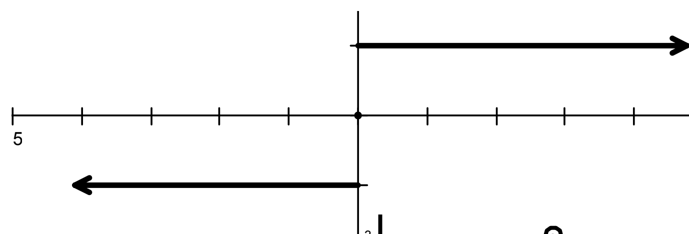
The stated domain includes only cases b) and c).

b)→ $x\left(\dfrac{1+(-1)}{2}\right) = 0$

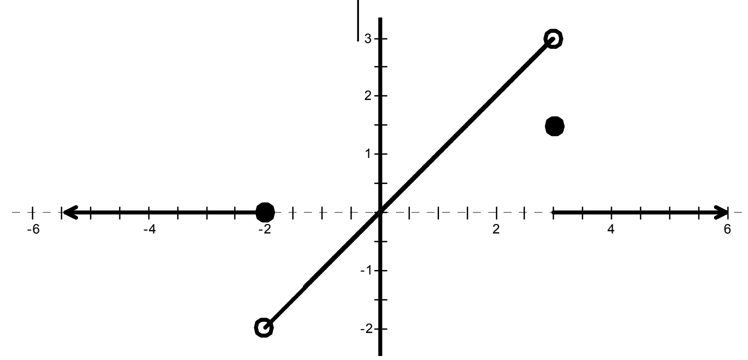
c) for $x = -2$ → $-2\left(\dfrac{1+0}{2}\right) = -1$ c) for $x = 3$ → $3\left(\dfrac{1+0}{2}\right) = 3/2$

and the range is **<u>{0, -1, 3/2}</u>**

The graph of sgn(x) is:



The graph of $f(x)$ over <u>all</u> reals is:



C) After t seconds, $(40-4t)(30-3t) = 108 + (4+2t)(3+6t)$ → $1200 - 240t + 12t^2 = 120 + 30t + 12t^2$

At this time, the sum of their perimeters is $2(70-7t) + 2(7+8t) = 154 + 2t$ → $154 + 8 =$ **<u>162</u>** feet

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008 SOLUTION KEY

Round 2

A) The powers of 7 and 3 are of cyclical order 4, i.e. $3^4 \equiv 1 \bmod 4$ and $7^4 \equiv 1 \bmod 4$
In other words, the rightmost digit of each product is 1.
Thus, 3 (and 7) raised to a power that is a multiple of 4 has a rightmost digit of 1.

$7^{218} \cdot 3^{507} = (7^{216} \cdot 3^{504}) 7^2 3^3 = (\_1)(\_1)(49)((27) = (\_3)$ → units digit = **<u>3</u>**

B) $x^2 - y^2 = (x + y)(x - y) = 31$.
Since the only possible factorization of a prime number is itself times 1,
we have $x + y = 31$ and $x - y = 1$ → $x = 16$ and $y = 15$.
Thus, $N = 16(15) = 2^4 \cdot 3 \cdot 5$. Since a factor of this product is only divisible by the prime factors 2, 3 and 5, the factor has the form $2^a 3^b 5^c$. Clearly, the largest possible value of a is 4 and the smallest is 0 (not 1). For 3 and 5, $(b_{max}, b_{min}) = (c_{max}, c_{min}) = (1, 0)$.
Thus, the total number of positive factors is (5)(2)(2) = **<u>20</u>**.

Note: Once you have the <u>prime</u> factorization of an integer, the number of positive integer factors depends only on the exponents. In this case, it was $(a + 1)(b + 1)(c + 1)$.

C) To be divisible by 15, an integer must be divisible by both 3 and 5.
Thus, the rightmost digit must be 5 and the sum of all the digits used must be divisible by 3.
The only two-digit possibility is 75.
The 3-digit possibilities can only be formed using {3, 5, 7} → 375 or 735
There are no 4-digit possibilities since the sum of the 4 digits is 17. → sum = **<u>1185</u>**

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008 SOLUTION KEY

Round 3

A) $\dfrac{1}{\sin 2\theta}+\dfrac{\cos 2\theta}{\sin 2\theta}=\dfrac{1+\cos 2\theta}{\sin 2\theta}=\dfrac{1+2\cos^2\theta-1}{2\sin\theta\cos\theta}=\dfrac{\cos\theta}{\sin\theta}$ = cot(θ) = 1

→ θ= **<u>45°, 225°</u>**

B) Using the double angle formula, sin(x) = 1 – 2cos^{2}40° = -(2cos^{2}40° – 1) = -cos(80°) = -sin(10°)
The related values of 10° in quadrants II, III and IV are 170°, 190° and 350°.
Since sin(x) is negative only in quadrants III and IV, x = **<u>190° or 350°</u>**.

C) Let A = *Arc*cos(-n/11) and B = $Arc\tan\left(-1/\left(2\sqrt{6}\right)\right)$.

Then $\pi/2 < A < \pi$ (quadrant 2) and $-\pi/2 < B < 0$ (quadrant 4)

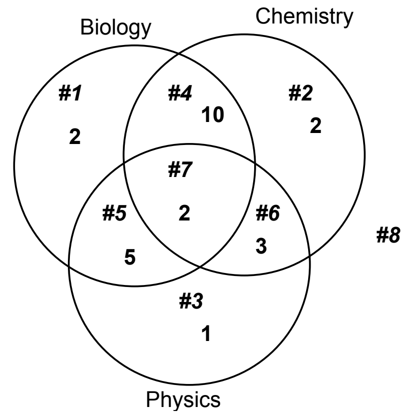
and sin(A + B) = sin$A$cos$B$ + sin$B$cos$A$ = $\dfrac{\sqrt{121-n^2}}{11}\cdot\dfrac{2\sqrt{6}}{5}+\dfrac{-1}{5}\cdot\dfrac{-n}{11}=\dfrac{2\sqrt{6}\sqrt{121-n^2}+n}{55}$

Thus, $2\sqrt{6}\sqrt{121-n^2}+n=53$ and the radicand 121 – n^2 must be 6 times a perfect square
Additionally, since n/11 is a cosine value, the only possible integer values of n are 1 ... 11.
Only n = **<u>5</u>** satisfies both conditions ($2\sqrt{6}\sqrt{96}+5=2\cdot 6\cdot 4+5=53$).

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008 SOLUTION KEY

Round 4

A) Using a Venn Diagram to separate the overlaps,



Biology	#1,4,5,7
Chemistry	#2,4,6,7
Physics	#3,5,6,7
Chemistry and Physics	#6,7
Biology and Physics	#5,7
Biology and Chemistry	#4,7
All 3 courses	#7

None = 35 – (2+10+2+5+2+3+1) = **<u>10</u>**

B) Let x denote the percent of the job completed. Then $\dfrac{3\cdot 4}{80} = \dfrac{1\cdot\frac{3}{4}}{x} \rightarrow 12x = 60 \rightarrow x = \underline{\mathbf{5}}$

C) Let z denote the mass of each cylinder and (x, y) the distances of pans A and B from the balance point respectively.
Equating the clockwise and counterclockwise torques keeps the system in equilibrium.
Thus, $8zx = 9y \rightarrow y = \dfrac{8zx}{9}$ and $25x = 2zy \rightarrow y = \dfrac{25x}{2z}$
Equating and canceling the x's in the numerator (since $x \neq 0$)
$\rightarrow \dfrac{8z}{9} = \dfrac{25}{2z} \rightarrow z^2 = \dfrac{225}{16} \rightarrow z = \underline{\mathbf{\dfrac{15}{4}\ (or\ 3\dfrac{3}{4},\ 3.75)}}$

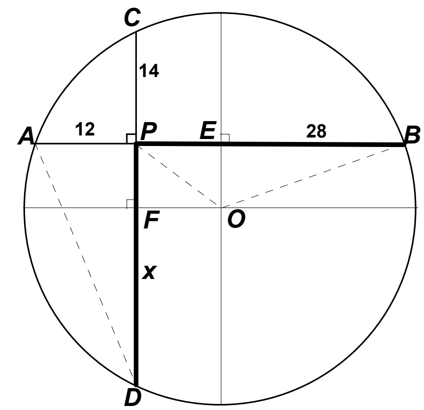
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008 SOLUTION KEY

Round 5

A) A point on the circumference moves 24π inches per second.

Converting, $24\pi \dfrac{\cancel{\text{in}}}{\cancel{\text{sec}}} \cdot \dfrac{1\,\text{ft}}{12\ \cancel{\text{in}}} \cdot \dfrac{60\ \cancel{\text{sec}}}{1\ \text{min}} = \underline{\boldsymbol{120\pi}}$ ft/min

B) $(5x-3)^2 = (3x-1)(10x-12) \rightarrow 25x^2 - 30x + 9 = 30x^2 - 46x + 12$
$\rightarrow 5x^2 - 16x + 3 = (5x-1)(x-3) = 0 \rightarrow x = 3$ [1/5 is extraneous]
$PA = 12$, $PB = 8$, $BC = 10$, $AC = 9$
Thus, the perimeter of $\Delta APC = (12 + 18 + 9) = \underline{\mathbf{39}}$.



C) Let $x = PD$. Applying the product-chord theorem, $14x = 12(28)$
$\rightarrow x = 24$. Since E and F are midpoints, $AE = 20 \rightarrow PE = 8$ and
$CF = 19 \rightarrow PF = OE = 5$.
Thus, in right ΔPEO, $PO^2 = 8^2 + 5^2 \rightarrow PO = \sqrt{89}$ and
in right ΔAPD, $AD^2 = 12^2 + 24^2 \rightarrow AD = 12\sqrt{5}$
$\rightarrow AD - PO = \underline{\mathbf{12\sqrt{5} - \sqrt{89}}}$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008 SOLUTION KEY

Round 6

A) $9 + 4d = 25 \rightarrow d = 4 \rightarrow t_{14} = t_{12} + 2(4) = \underline{\mathbf{57}}$

B) The first sequence is an <u>arithmetic</u> sequence with a common difference of 7, i.e. $t_n = 7n - 9$.
The second sequence is a <u>geometric</u> sequence with a common ratio of –2, i.e. $t_n = (3/8)(-2)^n$

The 10^{th} term in the geometric sequence is $(3/8)(-2)^{10} = 384$.
$7n - 9 > 384 \rightarrow n > 393/7 = 56+ \rightarrow n = 57 \rightarrow 7(57) - 9 = \underline{\mathbf{390}}$

C) Using the recursive part of the definition, $A_{N+2} = 2A_{N+1} + 3A_N$

$N = 3 \rightarrow A_5 = 2A_4 + 3A_3$

$N = 2 \rightarrow A_4 = 2A_3 + 3A_2$

Substituting for A_2 and A_5, $\begin{cases} 17 = 2A_4 + 3A_3 \\ A_4 = 2A_3 + 12 \end{cases} \rightarrow (A_3, A_4) = (-1, 10)$

$A_3 = 2A_2 + 3A_1 \rightarrow -1 = 8 + 3A_1 \rightarrow A_1 = -3$

$A_6 = 2A_5 + 3A_4 = 2(17) + 3(10) = 64$

Thus, $A_1 + A_6 = \underline{\mathbf{61}}$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008 SOLUTION KEY

Team Round

A) $g(x) = Ax^3 \rightarrow f(g(x)) = 8(Ax^3) = 27x^3 \rightarrow A = 27/8$

$h(x) = Bx^5 \rightarrow g(h(x)) = \frac{27}{8}(Bx^5)^3 = \frac{27B^3}{8}x^{15} \rightarrow f(g(h(x))) = 27B^3x^{15} = 27x^{15} \rightarrow B = 1$

$g(x) = \frac{27}{8}x^3 \rightarrow g^{-1}(x) = \left(\frac{8x}{27}\right)^{1/3} = \frac{2}{3}x^{1/3}$ and $h(x) = x^5 \rightarrow h^{-1}(x) = x^{1/5}$

$\therefore (1024)^{1/5} \cdot (2/3)(-1)^{1/3} = 4(2/3)(-1) = \underline{\mathbf{-\frac{8}{3}}}$

B) The prime factorization of $N! = 2^x5^y$(a bunch of irrelevant primes), where $x > y$ since every other factor is even and only every 5^{th} is a multiple of 5. Thus, the number of factors of 10 is determined by y. Let $[A]$ denote the largest integer less than or equal to A.

$\left[\frac{N}{5}\right]$ represents the number of multiples of 5 in the first N positive integers.

Each contains one factor of 5. However, some may contain more than one.

$\left[\frac{N}{25}\right]$ represents the number of multiples of 25 in the first N positive integers.

Each contains two factors of 5. However, some may contain more than 2.

By adding, $\left[\frac{N}{5}\right] + \left[\frac{N}{25}\right] + \left[\frac{N}{125}\right] + \left[\frac{N}{625}\right] + \ldots$, we eliminate any duplication and count the exact number of 5s in the integers from 1 to N inclusive.

Note that eventually a term will become 0 and all subsequent terms will be 0 insuring that even though there are an infinite number of terms, a finite total will always exist.

$1000! \rightarrow \left[\frac{1000}{5}\right] + \left[\frac{1000}{25}\right] + \left[\frac{1000}{125}\right] + \left[\frac{1000}{625}\right] + \ldots = 200 + 40 + 8 + 1 + 0 + 0 + \ldots = 249$ zeros

Therefore, 8000! would be expected to end in about 2000 zeros.

Specifically, 8000! ends in 1998 zeros. ($8000 \rightarrow 1600 + 320 + 64 + 12 + 2 + 0 = 1998$)

8004! ends in the same number of zeros, since none of the additional factors of 8001, 8002, 8003 and 8004 introduce any more factors of 5. Additional factors of 5 are introduced by each of these 9 factors : 8005, 8010, 8015, 8020, 8025, 8030, 8035, 8040 and 8045.

Since 8025 is a multiple of 25, it introduces 2 factors of 5.

Thus, N = **<u>8045</u>** is the smallest value for which $N!$ ends in exactly 2008 zeros.

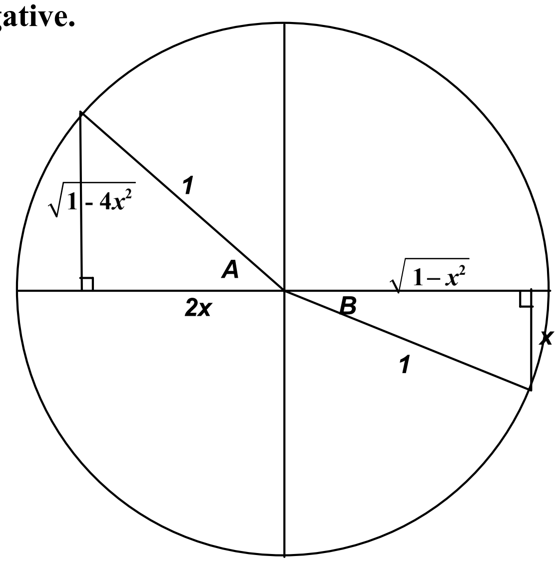
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008 SOLUTION KEY

Team Round – continued

C) Let $A = \text{Cos}^{-1}(2x)$ and $B = \text{Sin}^{-1}(x)$.

It was not necessary to specify that x must be negative.

[For $x = 0$, $A - B = \pi/2 - 0 = \pi/2 < 5\pi/6$
Positive values of x must be $\leq \frac{1}{2}$.
As x increases from 0 to $\frac{1}{2}$,
$\text{Cos}^{-1}(2x)$ decreases from $\pi/2$ to 0 and
$\text{Sin}^{-1}(x)$ increases from 0 to $\pi/6$.
Thus, there are <u>no</u> positive values for which the difference can be $5\pi/6$.]



For $x < 0$, A is in quadrant 2 ($\pi/2 < A < \pi$) and B is in quadrant 4 ($-\pi/2 < B < 0$) as indicated in the diagram at the right
Taking the sin of both sides,
$\text{Sin}(A - B) = \sin A \cos B - \sin B \cos A = 1/2$

$\rightarrow \sqrt{1-4x^2} \cdot \sqrt{1-x^2} - (2x)(x) = 1/2$

$\rightarrow \left((1-4x^2)\cdot(1-x^2)\right) = (1/2 + 2x^2)^2$

$\rightarrow 1 - 5x^2 + 4x^4 = 1/4 + 2x^2 + 4x^4$

$\rightarrow 7x^2 = 3/4 \rightarrow x^2 = \dfrac{3\cdot 7}{4\cdot 7\cdot 7} \rightarrow x = \underline{\mathbf{-\dfrac{\sqrt{21}}{14}}}$ (the positive root is rejected)

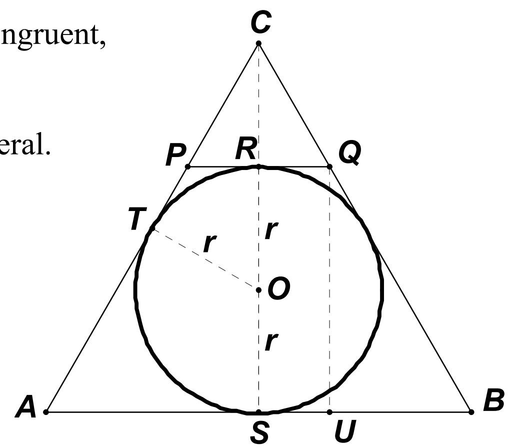
D) $(18+c) + \dfrac{c}{2} = (18-c)H \rightarrow 36 + 2c + c = 2(18-c)H \rightarrow H = \dfrac{36+3c}{36-2c} = 1 + \dfrac{5c}{36-2c}$

Therefore, the positive integer possibilities for c are 1 … 17.
$c = 1 \ldots 17 \rightarrow 8,\ 1 + 40/20 =$ **<u>(8, 3)</u>**; 12, $1 + 60/12 =$ **<u>(12, 6)</u>**; 16, $1 + 80/4 =$ **<u>(16, 21)</u>**
Other values of c produce fractional values of H.

Team Round – continued

E) Let $CS = h$
Then the facts that $\Delta PQC \sim \Delta ABC$ and $PQ : AB = 1 : 3$
$\rightarrow CR = h/3$, $RS = 2h/3$ and, therefore $r = h/3$

Let $PR = x$. Then $AS = 3x$
Since tangents to a circle from a common external point are congruent, $PT = x$ and $AT = 3x$
To maintain the 1 : 3 ratio for $CP : CA$, $CP = 2x$
Thus, since all sides of ΔABC have length $6x$, ΔABC is equilateral.

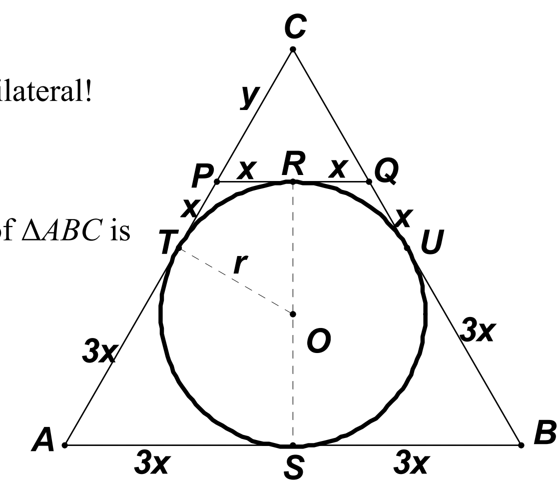


ΔCPR = 30-60-90 triangle and $CR = h/3 \rightarrow PR = \dfrac{h\sqrt{3}}{9}$

and the bases of the trapezoid are $\dfrac{2h\sqrt{3}}{9}$ and $\dfrac{2h\sqrt{3}}{3}$

$$\frac{\pi\left(\frac{h}{3}\right)^2}{\frac{1}{2}\cdot\frac{2}{3}h\left(\frac{2h\sqrt{3}}{9}+\frac{2h\sqrt{3}}{3}\right)} = \frac{\frac{\pi}{9}h^2}{\frac{1}{3}\left(\frac{8\sqrt{3}}{9}\right)h^2} = \frac{\pi}{9}\cdot\frac{27}{8\sqrt{3}} = \frac{3\pi}{8\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \underline{\boldsymbol{\frac{\pi\sqrt{3}}{8}}}$$

Easier Alternate Method:
Convince yourself that ΔABC is not only isosceles, it's equilateral!
Here's why!
$\overline{PQ} \parallel \overline{AB} \rightarrow \Delta PQC \sim \Delta ABC$
Therefore, $\dfrac{y}{y+4x} = \dfrac{2x}{6x} = \dfrac{1}{3} \rightarrow y = 2x$ and since each side of ΔABC is $6x$, it is equilateral.



The required ratio is $\dfrac{\pi r^2}{\frac{1}{2}(2r)(2x+6x)} = \dfrac{\pi r}{8x}$

Draw $\overline{OP}$. ΔOPT is a 30-60-90 right triangle
Thus, $r = x\sqrt{3}$.

Substituting and canceling, $\dfrac{\pi r}{8x} = \underline{\boldsymbol{\dfrac{\pi\sqrt{3}}{8}}}$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008 SOLUTION KEY

Team Round – continued

F) $\dfrac{Mn + \dfrac{n(n+1)}{2}}{n} = A \rightarrow M + \dfrac{n+1}{2} = \text{A} \rightarrow 2M + n + 1 = 2A$

$M : A = 2 : 3 \rightarrow 2A = 3M$ Substituting, $2M + n + 1 = 3M \rightarrow n = M - 1$

$n > 1{,}000{,}000 \rightarrow M > 1{,}000{,}001 \rightarrow$ min $M =$ **1,000,002**