MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2008
ROUND 1 ALG 2: SIMULTANEOUS EQUATIONS AND DETERMINANTS

ANSWERS

A) ______________________________

B) ______________________________

C) (pizza, sub, spag) = (\$___ , \$___ , \$___)

A) The system $\begin{cases} y = 2x + 1 \\ Ax + By = -9 \end{cases}$ represents a pair of perpendicular lines that intersect at $\left(\frac{1}{2}, 2\right)$.
Determine the ordered pair (A, B).

B) Find all real valuesof x for which $\begin{vmatrix} 1 & x & -3 \\ 2 & 0 & -1 \\ -3 & 2x-3 & x \end{vmatrix} = -7$

C) After the math meet, the bus stopped at the Pythagoras House of Pizza.
The mathletes noticed the following:
3 pizzas and 2 meatball subs cost \$4 more than 5 spaghetti with meat sauce.
Twice the cost of "one meatball sub and one spaghetti with meat sauce" would be \$2 less than 3 pizzas.
The combined cost of one meatball sub and one spaghetti with meat sauce is \$3 more than the cost of one pizza. Find the cost of each item.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008
ROUND 2 ALG1: EXPONENTS AND RADICALS

ANSWERS

A) ________________________________

B) ________________________________

C) (______________ , ______________)

A) Given: $N = 2x^{-2/3}$
If $x = 64$, find y, where $N = 4^y$.

B) Simply $\sqrt{4+\left(x-\frac{1}{x}\right)^2}\cdot\left(\frac{x}{3}+\frac{1}{3x}\right)^{-1}$ so that your answer is free of radicals and/or negative exponents.

C) Determine the ordered pair of positive integers (A, B) for which the quotient $\frac{\sqrt{49-8\sqrt{3}}}{\sqrt{21+12\sqrt{3}}}$ may be expressed as $\frac{A-B\sqrt{3}}{3}$.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2008
ROUND 3 ALG 2: POLYNOMIAL FUNCTIONS

ANSWERS

A) _______________________________

B) _______________________________

C) _______________________________

A) $x^3 + kx^2 - 13x + 6$ divided by $x - 2$ produces a remainder of k. Determine the value of k.

B) When the polynomial $p(x)$ is divided by $(x - 1)$, the quotient is Q(x) and the remainder is 1. When p(x) is divided by $(x + 1)$, the remainder is 7. Find $Q(-1)$

C) Find the cubic <u>equation</u> whose roots are 1 less than the reciprocals of the roots of

$$2x^3 - 5x^2 - 4x + 3 = 0.$$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008
ROUND 4 ALG 1: ANYTHING

ANSWERS

A) ________________________________

B) ________________________________

C) ________________________________

A) Solve for x. $$3 = 1 + \cfrac{2}{3 + \cfrac{1}{x}}$$

B) On a true false test, Claude, who is never prepared for tests, guessed the answers to all the questions and managed to get 24 out of the first 75 questions correct. Thereafter, he answered 2 out of every 5 correctly. Overall he answered 35% of the questions correctly. How many questions were there on the test?

C) Find all ordered pairs of positive integers (x, y) such that $2x - 5y = 1$ and $10 < x + y < 20$.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008
ROUND 5 PLANE GEOMETRY: ANYTHING

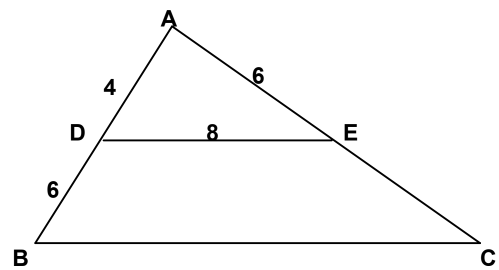
ANSWERS

A) ________________________________

B) ________________________________

C) ________________________________

A) Given ΔABC, $AD = 4$, $AE = 6$, $DE = 8$, $DB = 6$ and $\overline{DE} \parallel \overline{BC}$
Find BC. Diagram is not drawn to scale.



B) A square wire frame encloses 4 congruent circular disks each tangent to two adjacent sides of the frame and to two of the other disks. A second square wire frame has its vertices at the centers of these 4 disks. If the area of the region inside the smaller wire frame not covered by any of these disks is $12 - 3\pi$, what is the area of the region bounded by the two wire frames?

C) In a regular octagon $ABCDEFGH$, $AF = 8$, compute the area of ΔAFC.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008
ROUND 6 ALG 2: PROBABILITY AND THE BINOMIAL THEOREM

ANSWERS

A) ______________________________

B) ______________________________

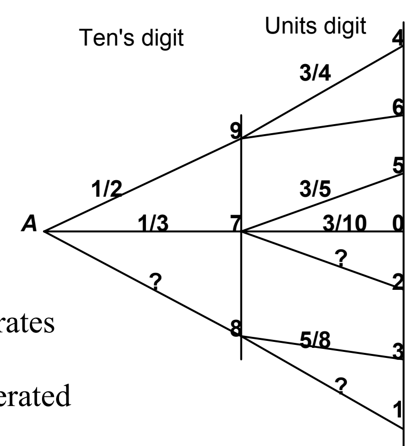
C) ______________________________

A) A container has 5 blue balls and 4 red balls. If 3 balls are simultaneously chosen at random, what is the probability that one is blue <u>and</u> two are red?

B) $(3 + 2i)^5 = a + bi$, where a and b denote real numbers and $i = \sqrt{-1}$. Compute $\sqrt{b-1}$.

C) Given:

The ten's digit must be 9, 7 or 8.
The units digit must be
4 or 6, if the ten's digit is 9
5, 0 or 2, if the ten's digit is 7
3 or 1, if the ten's digit is 8.



The graph at the right, starting at A and traveling to the right, generates some, but not all, two-digit integers with unequal probabilities of formation. For example, the probability that the number 94 is generated is $(1/2)(3/4) = 3/8$.

What is the probability that the number generated is divisible by 3?

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008
ROUND 7 TEAM QUESTIONS

ANSWERS

A) ____________________________ D) ____________________________

B) ____________________________ E) ____________________________

C) ____________________________ F) ____________________________

***** NO CALCULATORS ON THIS ROUND *****

A) In a magic square the sum in every column, in every row and along both diagonals is the same. Find the minimum value of x for which the following is a 3 x 3 magic square of positive integers.

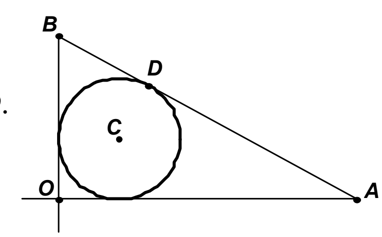
x	27	_
15	_	_
_	_	21

B) Let $P = \sqrt[3]{C} \cdot \sqrt[6]{C} \cdot \sqrt[12]{C} \cdot \sqrt[24]{C} \cdot \ldots$, where C and P are both integers.
If $C > 10^6$ and $N = P - 10^4 > 10^3$, determine the minimum value of N.

C) Let $P(x)$ denote the quadratic polynomial $Ax^2 + Bx + C$ with integer coefficients and $A > 0$. When $P(x)$ is divided by $x - h$, the remainder is k, but when $P(x)$ is divided by $x - k$, the remainder is h. If h and k are positive primes and $h + k = 9$, determine the minimum value of C.

D) Three clocks A, B and C are in serious need of a technician, who unfortunately is on vacation.
A has been ringing every 2 minutes and 30 seconds.
B has been ringing every 3 minutes and 20 seconds.
C has been ringing every 4 minutes and 10 seconds.
If it's now 2:15 PM and all three clocks just rang simultaneously, how many times earlier today did all three clocks ring together?

E) Right triangle ABO has circle C inscribed in it.
$A(24, 0)$, $B(0, 18)$ and $O(0, 0)$. Circle C intersects the hypotenuse $\overline{AB}$ at D.
Find the coordinates of point D.



F) In how many ways can you walk up a stairway that has 10 steps if you must take 1 or 2 steps at a time?

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2008 ANSWERS

Round 1 Alg 2: Simultaneous Equations and Determinants

A) (-2, -4)	B) 2, -11/2	C) ($8, $5, $6)

Round 2 Alg 1: Exponents and Radicals

A) -3/2	B) ±3 (both answers required)	C) (27 , 14)

Round 3 Alg2: Polynomial Functions

A) 4	B) -3	C) $3x^3 + 5x^2 - 4x - 4 = 0$ $(x - 1)(x + 2)(3x + 2) = 0$ or equivalent

Round 4 Alg 1: Anything

A) $-\frac{1}{2}$	B) 120	C) (8, 3) and (13, 5)

Round 5 Plane Geometry: Anything

A) 20	B) 36	C) $16\sqrt{2}$

Round 6 Alg 2: Probability and the Binomial Theorem

A) $\frac{5}{14}$	B) 11	C) $\frac{101}{240}$

Team Round

A) 7	D) 17
B) 1025	E) (9.6, 10.8)
C) 23	F) 89

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2008 SOLUTION KEY

Round 1

A) The slope of the first line is 2 → slope of the second line = $-\dfrac{A}{B} = -\dfrac{1}{2}$ → $B = 2A$

Thus, $A\left(\dfrac{1}{2}\right) + 2A(2) = -9$ → $A = -2$ → $(A, B) =$ **<u>(-2, -4)</u>**

B) Determining the determinant using the weaving technique,

$$\begin{matrix} 1 & x & -3 & 1 & x \\ 2 & 0 & -1 & 2 & 0 \\ -3 & 2x-3 & x & -3 & 2x-3 \end{matrix} \rightarrow [1(0)x + x(-1)(-3) + -3(2)(2x-3)] - [-3(0)(-3) + (2x-3)(-1)(1) + x(2)(x)]$$

→ $3x - 12x + 18 - 2x^2 + 2x - 3 = -7$ → $2x^2 + 7x - 22 = (2x + 11)(x - 2) = 0$ → **<u>-11/2, +2</u>**

C) Let (x, y, z) denote the cost of (pizza, sub, spaghetti)

Then (a) $3x + 2y = 4 + 5z$ → $3x + 2y - 5z = 4$,
(b) $2(y + z) = 3x - 2$ → $-3x + 2y + 2z = -2$
(c) $x = y + z - 3$ → $x - y - z = -3$

(a) – (b) → $4y - 3z = 2$ → $4y - 3z = 2$

3(c) + (b) → $-y - z = -11$ → $-4y - 4z = -44$ → $-7z = -42$ → $z = 6$, $y = 5$ and $x = 8$ → **<u>($8, $5, $6)</u>**

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2008 SOLUTION KEY

Round 2

A) Suppose $N = 4^{y}$.

$N = 2(64)^{-2/3} = 2(4)^{-2} = 2^{-3} = 4^{y} = 2^{2y} \rightarrow 2y = -3 \rightarrow y = \underline{\mathbf{-3/2}}$

B) $= \left(\sqrt{4 + x^2 - 2 + \dfrac{1}{x^2}}\right)\left(\dfrac{x^2+1}{3x}\right)^{-1} = \left(\sqrt{x^2 + 2 + \dfrac{1}{x^2}}\right)\left(\dfrac{3x}{x^2+1}\right) = \left(\sqrt{(x + x^{-1})^2}\right)\left(\dfrac{3x}{x^2+1}\right)$

$= \left(\sqrt{\dfrac{\left(x^2+1\right)^2}{x^2}}\right)\cdot\left(\dfrac{3x}{x^2+1}\right) = \dfrac{x^2+1}{|x|}\cdot\dfrac{3x}{x^2+1} = \dfrac{3x}{|x|} = \underline{\mathbf{\pm 3}}$

C) In order to extract the square roots in both the numerator and the denominator, the radicands must be perfect squares. Therefore, let $49 - 8\sqrt{3} = \left(A + B\sqrt{3}\right)^2$ and $21 + 12\sqrt{3} = \left(C + D\sqrt{3}\right)^2$.

Multiplying out and equating rational and irrational parts we have,

$\begin{cases} A^2 + 3B^2 = 49 \\ AB = 4 \end{cases} \rightarrow (A, B) = (-1, 4)$ and $\begin{cases} C^2 + 3D^2 = 21 \\ CD = 6 \end{cases} \rightarrow (C, D) = (3, 2)$

Thus, $\dfrac{\sqrt{49 - 8\sqrt{3}}}{\sqrt{21 + 12\sqrt{3}}} = \dfrac{-1 + 4\sqrt{3}}{3 + 2\sqrt{3}}\cdot\dfrac{3 - 2\sqrt{3}}{3 - 2\sqrt{3}} = \dfrac{-3 + 2\sqrt{3} + 12\sqrt{3} - 24}{-3} = \dfrac{27 - 14\sqrt{3}}{3} \rightarrow \underline{\mathbf{(27, 14)}}$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2008 SOLUTION KEY

Round 3

A)
$$\begin{array}{r|rrrr} & 1 & k & -13 & 6 \\ & & 2 & 2k+4 & 4k-18 \\ \hline 2 & 1 & k+2 & 2k-9 & 4k-12 \end{array} = k \rightarrow k = \underline{\mathbf{4}}$$

B) $p(\mathrm{x}) = (x - 1)Q(x) + 1$ and $p(\mathrm{x}) = (x + 1)Q_2(x) + 7$, where Q_2 is also unknown to us.
Substituting $x = -1$ in the second equation, $p(-1) = 7$. Even though Q_2 is unknown to us, it is being multiplied by zero!
Substituting, $x = -1$ in the first equation → $p(-1) = -2Q(-1) + 1$ → $7 = -2Q(-1) + 1$ → $Q(-1) = \underline{\mathbf{-3}}$

C) Method 1 (brute force):
Using synthetic substitution, find the roots of the original cubic equation

$$\begin{array}{r|rrrr} & 2 & -5 & -4 & 3 \\ \hline -1 & 2 & -7 & 3 & 0 \\ 3 & 2 & -1 & 0 & \\ 1/2 & 2 & 0 & & \end{array}$$

Thus, the original roots are -1, 3 and ½ and the corresponding roots of the new equation are -2, -2/3 and 1 producing factors of $(x + 2)$, $(3x + 2)$ and $(x - 1)$.
Expanding the product, the new equation is $(x + 2)(3x^2 - x - 2) = \underline{\mathbf{3x^3 + 5x^2 - 4x - 4 = 0}}$

Method II
The equation with reciprocal roots is $2(1/x)^3 - 5(1/x)^2 - 4(1/x) + 3 = 0$ and, multiplying through by x^3 to clear fractions, we have $3x^3 - 4x^2 - 5x + 2 = 0$. Notice that the coefficients have been reversed. Instead of replacing x by $(x - 1)$ and expanding, we'll again use synthetic substitution.

$$\begin{array}{r|rrrr} & 3 & -4 & -5 & 2 \\ \hline 1 & 3 & -1 & -6 & \boxed{-4} \\ 1 & 3 & 2 & \boxed{-4} & \\ 1 & 3 & \boxed{5} & & \\ 1 & \boxed{3} & & & \end{array}$$

→ $\underline{\mathbf{3x^3 + 5x^2 - 4x - 4 = 0}}$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008 SOLUTION KEY

Round 4

A) Subtracting 1 from both sides of the equation, $2 = \frac{2}{1} = \frac{2}{3+\frac{1}{x}}$

Since the numerators are equal, we can equate the denominators.

$1 = 3 + \frac{1}{x} \rightarrow \frac{1}{x} = -2 \rightarrow x = \underline{\boldsymbol{-\frac{1}{2}}}$

B) Let $5x$ denote the number of additional questions. Then

$\frac{24+2x}{75+5x} = 35\% = \frac{7}{20} \rightarrow 480 + 40x = 525 + 35x \rightarrow x = 9 \rightarrow$ # questions = 75 + 5(9) = **<u>120</u>**

C) $y = \frac{2x-1}{5} \rightarrow 10 < x + \frac{2x-1}{5} < 20 \rightarrow 50 < 7x - 1 < 100 \rightarrow 7.285^{\cdots} < x < 14.428^{\cdots}$

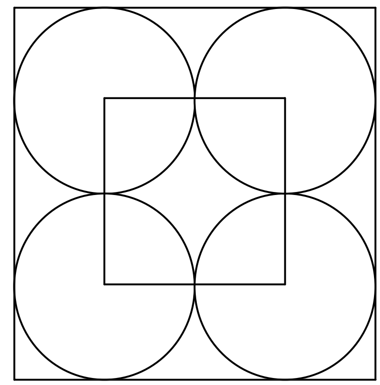
$\rightarrow x = 8, 9, \ldots 14$

Substituting, $x = 8$ immediately gives an integer result.

Since the first equation represents a line with a slope of 2/5, incrementing x by 5 gives the next x for which y is also an integer. Thus, **<u>(8, 3) and (13, 5)</u>**

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008 SOLUTION KEY

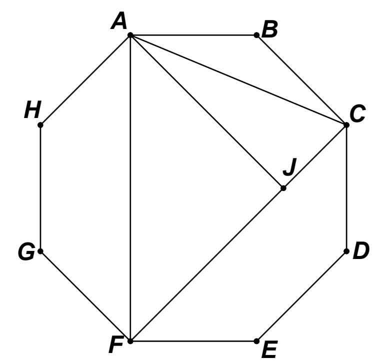
Round 5



A) $AB = 10$, $\Delta ADE \sim \Delta ABC \rightarrow \dfrac{AD}{AB} = \dfrac{DE}{BC} \rightarrow \dfrac{4}{10} = \dfrac{8}{BC}$

$\rightarrow 4(BC) = 8(10) \rightarrow BC = \underline{\mathbf{20}}$

B) $4r^2 - \pi r^2 = 12 - 3\pi \rightarrow r^2 = 3 \rightarrow \text{Area}_{\text{small square}} = 12$ and $\text{A}_{\text{large square}} = 48 \rightarrow$ required area = $\underline{\mathbf{36}}$



C) $AF = FC = 8$. The measure of an interior angle of a regular octagon is $\dfrac{6(180)}{8} = 135°$.

$m\angle EFC = m\angle GFA = 45°$, so $m\angle AFC = 45°$ also.

Draw $\overline{AJ} \perp \overline{FC}$. Then ΔAFJ is 45°-45°-90° and $AJ = 4\sqrt{2}$

Area of $\Delta AFC = \frac{1}{2}(FC)(AJ) = \frac{1}{2}(8)(4\sqrt{2}) = \underline{\mathbf{16\sqrt{2}}}$

Alternate method: $\text{Area}(\Delta AFC) = \dfrac{1}{2}(AF)(FC)\sin(\angle AFC)$

$$= \frac{1}{2}(8)(8)\sin(45°) = 32\cdot\frac{\sqrt{2}}{2} = \underline{\mathbf{16\sqrt{2}}}$$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008 SOLUTION KEY

Round 6

A) Pick one blue from the 5 available and two red from the 4 available without regard for order. The total number of [possible drawings is 3 from the 9 available also without regard for order. AND → MULTIPLY. Thus, this is simply a ratio of combinations.

$$p = \frac{\binom{5}{1}\cdot\binom{4}{2}}{\binom{9}{3}} = \frac{5\cdot 6}{\frac{9\cdot 8\cdot 7}{3\cdot 2\cdot 1}} = \frac{5\cdot 6}{3\cdot 4\cdot 5} = \underline{\mathbf{\frac{5}{14}}}$$

B) $(3+2i)^5 = 3^5 + 5\cdot 3^4(2i) + 10\cdot 3^3(2i)^2 + 10\cdot 3^2(2i)^3 + 5\cdot 3\left(2i\right)^4 + (2i)^5$
$= 243 + 405(2i) + 270(4i^2) + 90(8i^3) + 15(16i^4) + 32i^5$
$= 243 + 810i - 1080 - 720i + 240 + 32i$
→ $b = 122$ → $\sqrt{b-1} = \underline{\mathbf{11}}$

C) The number must be 96, 75, 72 or 81.
The probability of one of these numbers being generated is
$(1/2)(1/4) + 1/3(3/5) + (1/3)(1/10) + (1/6)(3/8) = 1/8 + 1/5 + 1/30 + 1/16$
$= (30 + 48 + 8 + 15)/240 =$ **<u>101/240</u>**

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008 SOLUTION KEY

Team Round

A) Let the unspecified entries in the square be denoted *a*, *b*, *c*, *d* and *e*.

Specifically, the square is $\begin{array}{|ccc|}\hline x & 27 & d \\ 15 & b & e \\ a & c & 21 \\ \hline\end{array}$

→

$$\begin{cases} \text{diagonals: } x+b+21=a+b+d \Rightarrow a=x-d+21 \\ \text{col1,row1: } x+15+a=x+27+d \Rightarrow d=a-12 \Rightarrow a=\dfrac{x+33}{2} \text{ and } d=\dfrac{x+9}{2} \\ \text{col1,row3: } x+15+a=a+c+21 \Rightarrow c=x-6 \\ \text{row1,col3: } x+27+d=d+e+21 \Rightarrow e=x+6 \\ \text{row2,col3: } 15+b+e=d+e+21 \to b-d=6 \to b-(a-12)=6 \to b=a-6 \Rightarrow b=\dfrac{x+21}{2} \end{cases}$$

Thus, the magic square becomes $\begin{array}{|ccc|}\hline x & 27 & \dfrac{x+9}{2} \\ 15 & \dfrac{x+21}{2} & x+6 \\ \dfrac{x+33}{2} & x-6 & 21 \\ \hline\end{array}$

Since $c = x - 6$ must represent a positive integer,

the minimum possible *x*-value is **7** → $\begin{array}{|ccc|}\hline 7 & 27 & 8 \\ 15 & 14 & 13 \\ 20 & 1 & 21 \\ \hline\end{array}$

B) $P = C^{\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\frac{1}{24}+\ldots}$ The exponent is an infinite geometric progression.

The sum is given by $\dfrac{a}{1-r} = \dfrac{\frac{1}{3}}{1-\frac{1}{2}} = \dfrac{2}{3}$ → $P = C^{\frac{2}{3}}$

Thus, *C* must be a perfect cube to insure that *P* is an integer. Let $C = x^3$, where $x > 10^2$.

$N = P - 10^4 = C^{\frac{2}{3}} - 10^4 = \left(x^3\right)^{\frac{2}{3}} - 10^4 = x^2 - 10^4 > 10^3$ and the minimum *x* is 101.

$101^2 = 10201, \ldots\ 104^2 = 10816$ fail, but $105^2 = 11025$ → $N =$ **1025**

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008 SOLUTION KEY

Team Round – continued

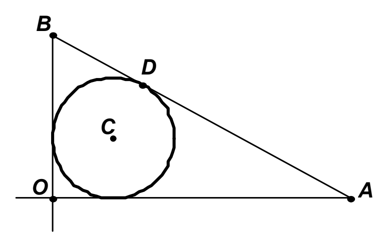
C) If the sum of two primes is odd, then one of the primes must be 2.
Therefore, $h = 2 \rightarrow k = 7$ (or vice versa)

$$\begin{cases} P(x) = Ax^2 + Bx + C \\ P(2) = 7 \\ P(7) = 2 \end{cases} \rightarrow \begin{cases} 4A + 2B + C = 7 \\ 49A + 7B + C = 2 \end{cases} \rightarrow 45A + 5B = -5 \rightarrow 9A + B = -1$$

$\rightarrow B = -1 - 9A$
Substituting, $4A + 2(-1 - 9A) + C = 7 \rightarrow -14A - 2 + C = 7 \rightarrow C = 14A + 9$
Since $A > 0$, the minimum value of C occurs when $A = 1$, i.e. $C =$ **<u>23</u>**.

D) A, B and C have been ringing every $\frac{5}{2}, \frac{10}{3}$ and $\frac{25}{6}$ minutes respectively, or in terms of a common denominator - $\frac{15}{6}, \frac{20}{6}$ and $\frac{25}{6}$ minutes.
The Least common multiple of (15, 20, 25) = 300
Thus, the bells ring together every 300/6 = 50 minutes.
The number of minutes since midnight is 14(60) + 15 = 855.
855/50 = 17.1 → **<u>17</u>** times
Note: 855 – 50(1), 855 – 50(2), … 855 – 50(17) = 5
The first simultaneous ringing occurred at 12:05 AM

E) $AB = 30$. The radius of circle $C = ½(24 + 18 – 30) = 6$
Thus, C is located at (6, 6). The equation of $\overline{AB}$ (m = -18/24 = -3/4): $3x + 4y = 72$
$\overline{CD} \perp \overline{AB}$, $m_{CD} = 4/3 \rightarrow \text{Eqtn}_{CD}$: $4x - 3y = 6$



Using Cramer's rule: $x = \dfrac{-240}{-25} =$ **<u>9.6</u>** and $y = \dfrac{18-288}{-25} =$ **<u>10.8</u>**

Note: Let $M(0, 6)$ and $N(6,0)$ Then area (ΔMND) =

$$\frac{1}{2}\begin{vmatrix} 0 & 6 \\ 6 & 0 \\ 9.6 & 10.8 \\ 0 & 6 \end{vmatrix} = \frac{64.8+57.6-36}{2} = 43.2$$

Distance from D to $\overline{MN}$: $d((9.6, 10.8), x + y = 6) = \dfrac{20.4-6}{\sqrt{2}} = 7.2\sqrt{2}$

Area(ΔMND) = $\dfrac{1}{2} \cdot 6\sqrt{2} \cdot 7.2\sqrt{2} = 43.2$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008 SOLUTION KEY

Team Round – continued

F) These are the combinations of 1 and 2 steps:

		# of different arrangements
11111 11111	→	1
2 1111 1111	→	$\frac{9!}{8!}$ or ${}_9C_1 = 9$
22 111 111	→	$\frac{8!}{2!6!}$ or ${}_8C_2 = 28$
222 11 11	→	$\frac{7!}{3!4!}$ or ${}_7C_3 = 35$
2222 1 1	→	$\frac{6!}{2!4!}$ or ${}_6C_2 = 15$
22222	→	1

Total: **<u>89</u>**