MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2009
ROUND 1 COMPLEX NUMBERS (No Trig)

******* NO CALCULATORS IN THIS ROUND *******

ANSWERS

A) ____________________________

B) ____________________________

C) ____________________________

Note: $i = \sqrt{-1}$

A) Simplify completely: $\dfrac{1+2i+3i^2+4i^3}{1-2i+3i^2-4i^3}$

B) Given: $(3+3i)^{40} = r^n$, where r and n are both integers
Determine the <u>smallest</u> possible value of the sum $r + n$.

C) If $\sqrt{-40-9i} = A + Bi$, compute $\left(\dfrac{A}{B}\right)^2$.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2009
ROUND 2 ALGEBRA 1: ANYTHING

***** NO CALCULATORS IN THIS ROUND *****

ANSWERS

A) _______________________________

B) _______________________________

C) _______________________________

A) Find 4 consecutive odd integers whose sum is 213 more than the largest of these integers.

B) If $x + y = 3$ and $xy = -10$, find the <u>largest</u> possible value of $\frac{x}{y}$.

C) A train travels 150 miles in w hours. If the rate of the train were increased by x mph, the train would arrive at its destination in 2 less hours. Find x in terms of w.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2009
ROUND 3 PLANE GEOMETRY: AREAS OF RECTILINEAR FIGURES

******* NO CALCULATORS IN THIS ROUND *******

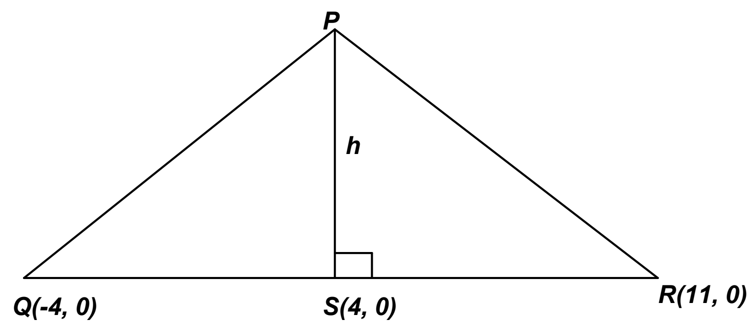
ANSWERS

A) _________________________ units^2

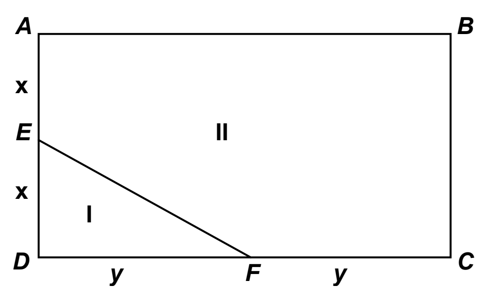
B) _________________________ units^2

C) _________________________ units

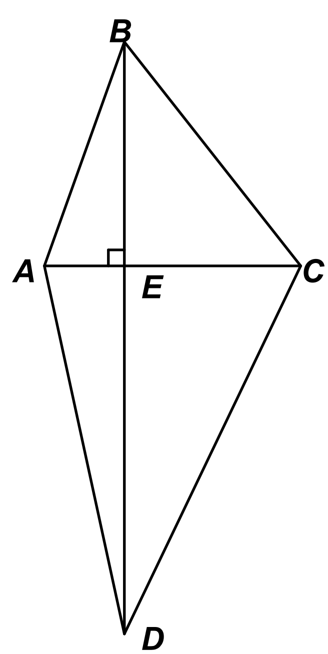
A) The area of ΔPQR is 45 square units.
The areas of ΔPQS and ΔPSR are unequal.
Determine the smaller of the two areas.



B) Rectangle $ABCD$ has an area of 500 square units.
E and F are midpoints of two adjacent sides.
Determine the area of the larger of the two regions inside $ABCD$ created by $\overline{EF}$.



C) Given: quadrilateral $ABCD$ with perpendicular diagonals and
$AB = 13$, $BC = 15$, $BD = 52$, $AC = 14$
To the nearest integer, what is the perimeter of ΔADE?



MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2009
ROUND 4 ALG 1: FACTORING AND ITS APPLICATIONS

******* NO CALCULATORS IN THIS ROUND *******

ANSWERS

A) ____________________________

B) ____________________________

C) ____________________________

A) Solve for x: $$\frac{x^2-10x+12}{10x-x^2-28}=\frac{1}{3}$$

B) Solve for x: $5x^2+4x-x^3-20=0$

C) The polynomial $x^{24}-x^8-256x^{16}+256$ can be written as the product of N binomial factors of the form $(x^a \pm b)$, where a and b are positive integers. Determine the <u>maximum</u> value of N.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2009
ROUND 5 TRIG: FUNCTIONS OF SPECIAL ANGLES

******* NO CALCULATORS IN THIS ROUND *******

ANSWERS

A) (________ , ________ , ________)

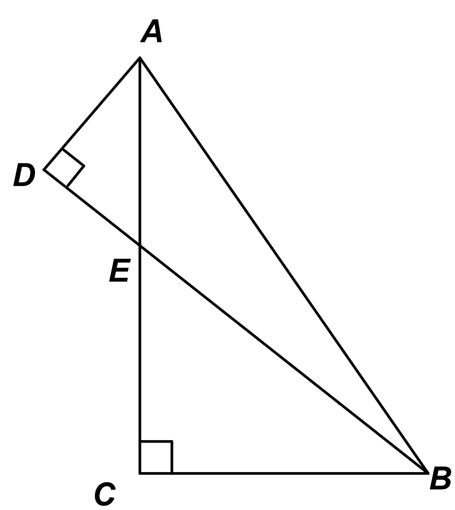
B) ________________________________

C) ________________________________

A) In simplest form, $(\tan 240° + \tan 405°)^3 = A + B\sqrt{C}$. Determine the ordered triple (A, B, C).

B) For the purpose of this question, suppose special angles denote angles belonging to the 30° family, 45° family, 60° family or the quadrantal family $\left(0° + 90k\right)$.
Compute $\tan(x)$ given that $2\tan(x) = 3\cot(x) - 1$ and x is not a special angle.

C) In ΔABC, $m\angle C = m\angle D = 90°$, $AB = 4$, $m\angle BAC = 30°$ and $BC = EC$.
Find BD in simplified radical form.



MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2009
ROUND 6 PLANE GEOMETRY: ANGLES, TRIANGLES AND PARALLELS

******* NO CALCULATORS IN THIS ROUND *******

ANSWERS

A) ______________________________

B) ______________________________

C) ______________________________ °

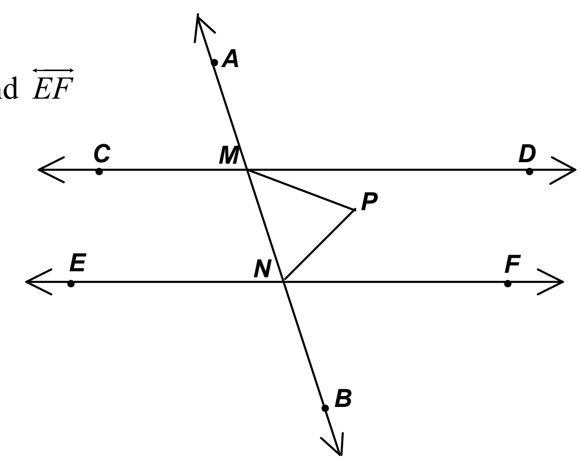
A) The measures of the vertex and base angles of an isosceles triangle are in a 4 : 3 ratio. If the vertex angle is the larger of these two angles, compute the measure of an exterior angle at the base.

B) A regular polygon has 740 diagonals. How many degrees in an exterior angle of this polygon?

C) $\overleftrightarrow{CD} \| \overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ is a transversal intersecting $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ in points M and N respectively.
P is a point between the parallel lines such that

$$m\angle NMP = 3m\angle PMD$$
$$m\angle MNP = 4m\angle PNF$$

If $m\angle AMD = (7x-40)°$ and $m\angle MNF = (5x)°$, find $m\angle P$.



MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2009
ROUND 7 TEAM QUESTIONS

***** CALCULATORS ARE PERMITTED IN THIS ROUND *****

ANSWERS

A) ____________________________ D) ____________________________________

B) ____________________________ E) ____________________________

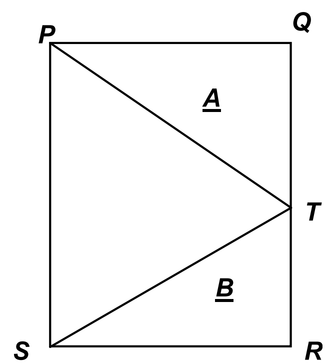
C) ____________________________ F) ____________________________

A) Given: $z = 1-\sqrt{3}i$ Compute: $\left|\sqrt{z}\cdot\sqrt[3]{z^2}\cdot\sqrt[6]{z^5}\right|$

B) A brick mason can do a job in 6 less hours than his apprentice. He and his apprentice work together for 4 hours. After the fourth hour, the apprentice works alone and finishes the remainder of the job in three hours. If the brick mason had been able to hire k apprentices, each of whom worked at the same rate as his original apprentice, and they all worked together with him from the start, the job would have been finished in a time of one hour or less.
What is the minimum possible value of k?

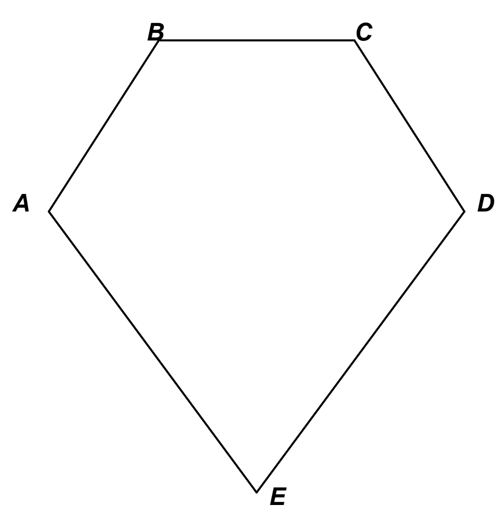
C) Point P is located in the interior of rectangle $ABCD$. (No diagram given.)
$AD = 52$, $PA = 56$, $PB = 25$ and $PC = 33$. Compute AB.

D) Factor completely over the integers. $a^{4x} - 4a^{3x} + a^{2x} + 6a^x$



E) $PQRS$ is a rectangle.
Let A and B denote the areas of ΔPQT and ΔSRT respectively.
If $m\angle PTS = 60°$ and $PT = \tan(\angle TPS) = 1$, compute $\dfrac{A}{B}$.

F) $ABCDE$ is a pentagon, $AB = BC = CD$ and $DE = EA$.
For integers d and k, $m\angle A = m\angle B = m\angle C = m\angle D = d°$ and $m\angle E$ is $5k°$.
Compute the largest possible value of $\dfrac{m\angle EAD}{m\angle DAB}$ that is less than 1.



MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2009 ANSWERS

Round 1 Algebra 2: Complex Numbers (No Trig)

A) i B) 38 C) $\frac{1}{81}$

Round 2 Algebra 1: Anything

A) 69, 71, 73, 75 B) $-\frac{2}{5}$ (or -0.4) C) $\frac{300}{w(w-2)}$ or equivalent

Round 3 Plane Geometry: Area of Rectilinear Figures

A) 21 B) 437.5 $\left(\text{or } \frac{875}{2}\right)$ C) 85

Round 4 Algebra 1: Factoring and its Applications

A) 2, 8 B) ± 2, 5 C) 9

Round 5 Trig: Functions of Special Angles [Non-Calculator Round]

A) (10, 6, 3) B) $-\frac{3}{2}$ C) $\sqrt{6}+\sqrt{2}$

Round 6 Plane Geometry: Angles, Triangles and Parallels

A) 126 B) 9 C) 40°

Team Round

A) 4

B) 13

C) $\frac{837}{13}$

D) $a^x\left(a^x-2\right)\left(a^x+1\right)\left(a^x-3\right)$

E) $2+\sqrt{3}$

F) $\frac{10}{13}$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2009 SOLUTION KEY

Round 1

A) $\dfrac{1+2i+3i^2+4i^3}{1-2i+3i^2-4i^3}=\dfrac{1+2i-3-4i}{1-2i-3+4i}=\dfrac{-2-2i}{-2+2i}=\dfrac{1+i}{1-i}\cdot\left(\dfrac{1+i}{1+i}\right)=\dfrac{1+i^2+2i}{1-i^2}=\dfrac{2i}{2}=\underline{\boldsymbol{i}}$

B) $=3^{40}(1+i)^{40}=3^{40}(2i)^{20}=3^{40}2^{20}i^{20}=3^{40}2^{20}(1)=9^{20}2^{20}=18^{20}$ ➔ $r+n=$ **<u>38</u>**

Note: $18^{20}=\left(18^2\right)^{10}=324^{10}$. Such equivalent expressions produce larger values of $r+n$.

C) If $z=A+Bi$, then $z^2=-40-9i=A^2+2ABi-B^2=\left(A^2-B^2\right)+2ABi$. But, $|z|=\sqrt{A^2+B^2}$

and $|z^2|=\sqrt{(-40)^2+(-9)^2}=\sqrt{41^2}=41$ (9 – 40 – 41 is a Pythagorean Triple.)

Since $|z|^2=|z^2|$, we have $A^2+B^2=41$.

Equating the real parts, the imaginary parts and the absolute values, we have these three conditions: .

$$\begin{cases}(1)\ A^2-B^2=-40\\(2)\ 2AB=-9\\(3)\ A^2+B^2=41\end{cases}$$

(2) ➔ A and B have <u>opposite</u> signs.

(1) + (3) ➔ $2A^2=1$, (3) – (1) ➔ $2B^2=81$ and $(A,B)=\left(\pm\dfrac{1}{\sqrt{2}},\mp\dfrac{9}{\sqrt{2}}\right)$ ➔ $\left(\dfrac{A}{B}\right)^2=\dfrac{\frac{1}{2}}{\frac{81}{2}}=\underline{\boldsymbol{\dfrac{1}{81}}}$

Proof of the fact that for any **<u>complex</u>** number, $|z|^2=|z^2|$.

Let $z=x+yi$. Then $z^2=(x+yi)^2=x^2+2xyi+y^2i^2=\left(x^2-y^2\right)+(2xy)i$

$$|z|^2=\left(\sqrt{x^2+y^2}\right)^2=x^2+y^2$$

$$|z^2|=\sqrt{\left(x^2-y^2\right)^2+(2xy)^2}=\sqrt{\left(x^4-2x^2y^2+y^4\right)+4x^2y^2}=\sqrt{x^4+2x^2y^2+y^4}$$

$$=\sqrt{\left(x^2+y^2\right)^2}=x^2+y^2$$ By the transitive property, $|z|^2=|z^2|$.

Round 2

A) Let the 4 numbers be x, $x+2$, $x+4$ and $x+6$. Then:
$4x+12=213+x+6$ ➔ $3x=207$ ➔ $x=69$ ➔ **<u>69, 71, 73, 75</u>**

B) By solving $x(3-x)=-10$ or judicious guess and check, $(x,y)=(5,-2)$ or $(-5,2)$.

The possible values of $\dfrac{x}{y}$ are –2.5 or –0.4 . The larger value is **<u>–0.4 .</u>**

C) Let R_2 denote the new rate and R_1 the original rate.

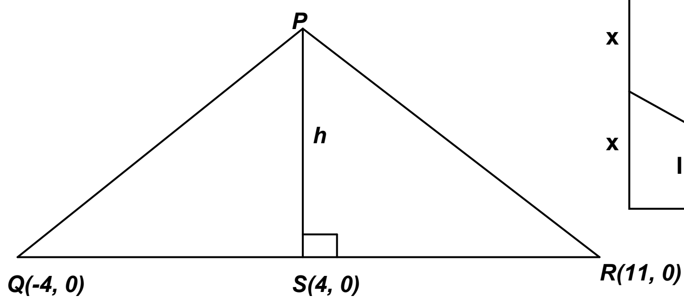
Since $R\cdot T=D$, we have $R_2=\dfrac{150}{w-2}$ and $R_1=\dfrac{150}{w}$ and $\dfrac{150}{w-2}=\dfrac{150}{w}+x$

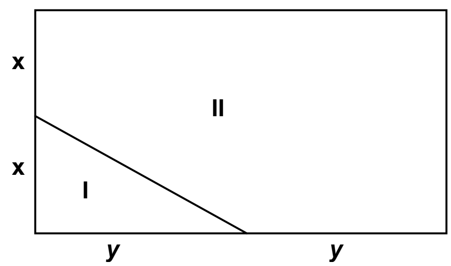
Clearing fractions, $150w=150(w-2)+x(w)(w-2)$ ➔ $150w=150w-300+xw^2-2xw$

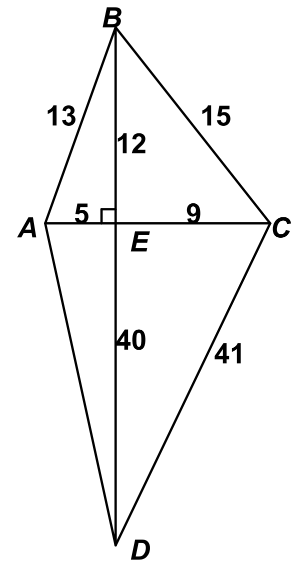
Canceling, we have $300=xw^2-2xw=x\left(w^2-2w\right)$ ➔ $x=\underline{\boldsymbol{\dfrac{300}{w^2-2w}}}$ or $\underline{\boldsymbol{\dfrac{300}{w(w-2)}}}$.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2009 SOLUTION KEY

Round 3







A) $QR = 15$, $QS = 8$ and $SR = 7$

$\frac{1}{2}(15)h = 45 \rightarrow h = 6$

ΔPSR has the smaller area, $\frac{1}{2}\cdot 7\cdot 6 = \underline{\mathbf{21}}$

B) Area(I) = $\frac{1}{2}xy$, Area(II) = $4xy - \frac{1}{2}xy = \frac{7}{2}xy$

Thus, regardless of the dimensions of the rectangle, region II has an area 7/8 that of the rectangle $\rightarrow \frac{7}{8}(500) = \underline{\mathbf{437.5}}$ or $\underline{\left(\mathbf{\frac{875}{2}}\right)}$

C) Noting special right triangles 5 - 12 - 13, 3(3 - 4 - 5) and 9 – 40 - 41, the problem is almost done.

$AD = \sqrt{1625} = 5\sqrt{65}$

65 is only slightly bigger than the perfect square 64.

$8.1^2 = 65.61 \rightarrow \sqrt{65} < 8.1 \rightarrow 5\sqrt{65} < 40.5$

Thus, to the nearest integer, the perimeter of ΔADE is **<u>85</u>**.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2009 SOLUTION KEY

Round 4

A) Cross multiplying, $\dfrac{x^2-10x+12}{10x-x^2-28}=\dfrac{1}{3}$ ➔ $3x^2-30x+36=10x-x^2-28$

➔ $4x^2-40x+64=4(x^2-10x+16)=4(x-2)(x-8)=0$ ➔ $x=$ **<u>2, 8</u>**

B) $5x^2+4x-x^3-20=\left(5x^2-x^3\right)+\left(4x-20\right)=x^2\left(5-x\right)-4(5-x)=0$ ➔ $\left(x^2-4\right)(5-x)=0$

➔ $x=$ **<u>±2, 5</u>**

C) $x^{24}-x^8-256x^{16}+256 = \left(x^{24}-256x^{16}\right)-\left(x^8-256\right)=\left(x^{16}-1\right)\left(x^8-256\right)=$

$\left(x^8+1\right)\left(\boldsymbol{x^8-1}\right)\left(x^4+16\right)\left(\boldsymbol{x^4-16}\right) = \left(x^8+1\right)\left(x^4+1\right)\left(\boldsymbol{x^4-1}\right)\left(x^4+16\right)\left(x^2+4\right)\left(\boldsymbol{x^2-4}\right) =$

$\left(x^8+1\right)\left(x^4+1\right)\left(x^2+1\right)(x+1)(x-1)\left(x^4+16\right)\left(x^2+4\right)\left(x+2\right)\left(x-2\right)$ ➔ ***N* = <u>9</u>**

Round 5

A) $= (\sqrt{3}+1)^3 = (\sqrt{3}+1)\,(\sqrt{3}+1)^2 = (\sqrt{3}+1)\,(2\sqrt{3}+4) = 10+6\sqrt{3}$ ➔ $(A, B, C) =$ **<u>(10, 6, 3)</u>**

B) $2\tan(x) = 3\cot(x) - 1$ ➔ $2\tan^2(x) = 3 - \tan(x)$

➔ $2\tan^2(x) + \tan(x) - 3 = (2\tan x + 3)(\tan x - 1)$ ➔ $\tan(x) = \boldsymbol{-\dfrac{3}{2}}$

(1 is extraneous since x would be special, i.e. it would belong to the 45° family.)

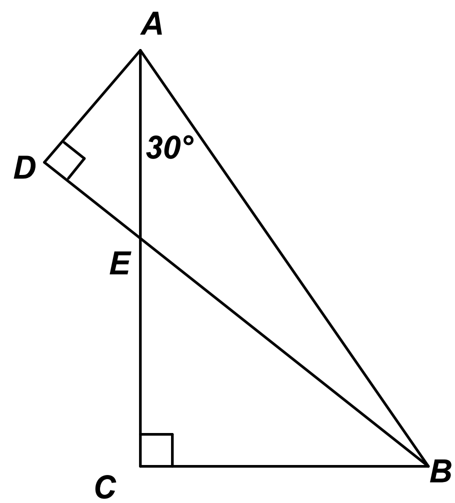
C) $m\angle CBA = 60°$, $BC = 2$, $AC = 2\sqrt{3}$

$m\angle CBE = m\angle CEB = m\angle AED = 45°$

$BE = 2\sqrt{2}$, $AE = 2\sqrt{3}-2$ and

$DE = \dfrac{2\sqrt{3}-2}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{2\sqrt{6}-2\sqrt{2}}{2}=\sqrt{6}-\sqrt{2}$

Thus, $BD = BE + ED = 2\sqrt{2}+\left(\sqrt{6}-\sqrt{2}\right) = \underline{\boldsymbol{\sqrt{6}+\sqrt{2}}}$



Alternate solution

In right ΔBAD, $\dfrac{BD}{AB}=\cos\left(\angle DBA\right)$.

$m\angle DBA = 60° - 45° = 15°$

Thus, $BD = 4\cos(15°) = 4\cos(45° - 30°) = 4\left(\cos 45°\cos 30° + \sin 45°\sin 30°\right)$

$= 4\left(\dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2}+\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}\right)=\underline{\boldsymbol{\sqrt{6}+\sqrt{2}}}$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2009 SOLUTION KEY

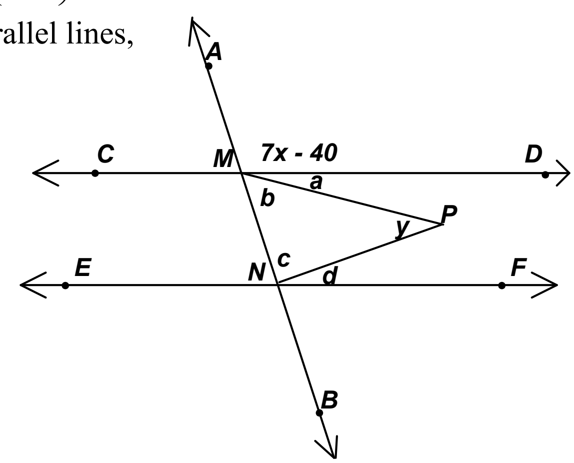
Round 6

A) $4x + 2(3x) = 180 \Rightarrow x = 18 \Rightarrow$ base angle: 54° $\Rightarrow$ exterior angle: **126°**

B) $n(n-3)/2 = 740 \Rightarrow n(n-3) = 1480$
Rather than trying to factor this quadratic by trial and error, guess at a value for n. If the result is too low, try a larger value of n; if the result is too high, try a smaller value of n.
$n = 35 \Rightarrow 35(32) = 1120$ (too low)
$n = 45 \Rightarrow 45(42) = 1890$ (too high)
Since 1120 is closer to 1480, we'll start at 40 and step down until we find n.

$n = 40 \Rightarrow 40(37) = 1480$ Bingo!
A regular polygon with 40 sides has exterior angle with $\left(\frac{360}{40}\right) = \underline{\mathbf{9}}°$

C) Since $\angle AMD$ and $\angle MNF$ are corresponding angles of parallel lines, we have $7x - 40 = 5x \Rightarrow x = 20$
$m\angle AMD = 100 \Rightarrow a + b = 80$ and $c + d = 100$
$m\angle NMP = 3m\angle PMD \Rightarrow b = 3a$
Thus, $(a, b) = (20, 60)$
$m\angle MNP = 4m\angle PNF \Rightarrow c = 4d$
Thus, $(c, d) = (80, 20)$
Finally, $y = 180 - (b + c) = 180 - 140 = \underline{\mathbf{40}}$



MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2009 SOLUTION KEY

Team Round

A) $\left|\sqrt{z}\cdot\sqrt[3]{z^2}\cdot\sqrt[6]{z^5}\right| = \left|z^{\frac{1}{2}}\cdot z^{\frac{2}{3}}\cdot z^{\frac{5}{6}}\right| = \left|z^{\frac{1}{2}+\frac{2}{3}+\frac{5}{6}}\right| = \left|z^{\frac{6+8+10}{12}}\right| = \left|z^2\right|$

$z^2 = 1-2\sqrt{3}i-3 = -2-2\sqrt{3}i \rightarrow \left|z^2\right| = \sqrt{4+12} = \underline{\mathbf{4}}$

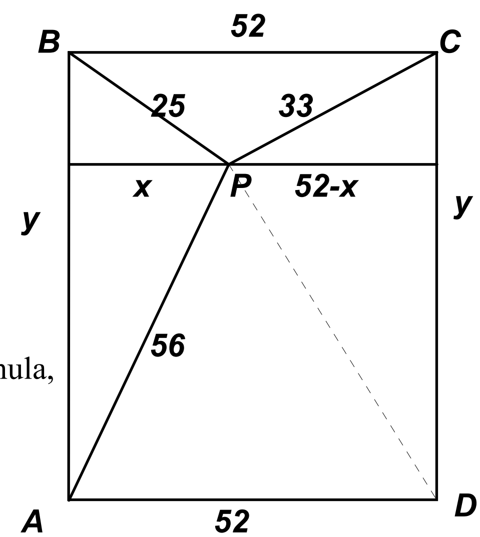
B) Let $\frac{1}{x}$ denote the rate at which the brick mason works, i.e. the fraction of the job he does in one hour. Then $\frac{4}{x}+\frac{(4+3)}{x+6}=1 \rightarrow 4(x+6)+7x = x^2+6x \rightarrow x^2-5x-24 = (x-8)(x+3) = 0$
$\rightarrow x = 8$ Thus, the mason and apprentice take 8 hours and 14 hours respectively to complete the job. Assume a minimum of A apprentices are needed

$\frac{1}{8}(1)+A\left(\frac{1}{14}\right)(1)\ge 1 \rightarrow \frac{A}{14}\ge\frac{7}{8} \rightarrow A\ge\frac{98}{8}=12.25 \rightarrow A_{\min} = \underline{\mathbf{13}}$

[12 apprentices and 1 brick mason take T hours to finish

$\frac{1}{8}T+12\left(\frac{1}{14}T\right)=1 \rightarrow \frac{T}{8}+\frac{6T}{7}=1 \rightarrow 7T+48T=56$

$\rightarrow T = 56/55 > 1$]



C) $25^2+PD^2 = 33^2+56^2 \rightarrow PD^2 = 3600$
$\rightarrow PD = 60$ (Refer to note on Contest 1 Round 2.) Using Heron's formula,

Area(ΔPBC) = $\sqrt{55(30)(22)(3)} = \sqrt{3^2\cdot 11^2\cdot 10^2} = 330$

Area(ΔPAD) = $\sqrt{84(28)(32)(24)} = \sqrt{2^{12}\cdot 3^2\cdot 7^2} = 1344$

Let $y = AB = CD$.

Area(rectangle) = $52y = 1674+\frac{1}{2}xy+\frac{1}{2}y(52-x) = 1674+26y \rightarrow 26y = 1674 \rightarrow y = AB = \underline{\mathbf{\frac{837}{13}}}$

D) Hoping to take advantage of a binomial of the form $\left(a^x+c\right)$, where c is a constant and thinking of Pascal's triangle:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$a^{4x}-4a^{3x}+a^{2x}+6a^x = a^{4x}-4a^{3x}+(6-5)a^{2x}+(10-4)a^x+(1-5+4)$

Regrouping, we have $\left(a^{4x}-4a^{3x}+6a^{2x}-4a^x+1\right)-5\left(a^{2x}+2a^x+1\right)+4$

$= \left(a^x-1\right)^4-5\left(a^x-1\right)^2+4 = \left(\left(a^x-1\right)^2-1\right)\left(\left(a^x-1\right)^2-4\right)$

$= \left(a^x-1+1\right)\left(a^x-1-1\right)\left(a^x-1+2\right)\left(a^x-1-2\right) = \underline{\boldsymbol{a^x\left(a^x-2\right)\left(a^x+1\right)\left(a^x-3\right)}}$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2009 SOLUTION KEY

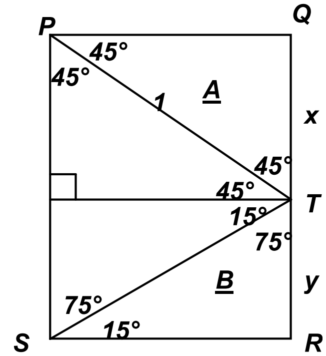
Team Round – continued

E) From the diagram at the right, we see that $x = \sin 45$ and

ΔPTS: $\dfrac{\sin 75}{1} = \dfrac{\sin 45}{ST}$ ➔ $ST = \dfrac{\sin 45}{\sin 75}$ $\quad \Delta TRS$: $\dfrac{y}{ST} = \sin 15$

$$y = \frac{\sin 15 \cdot \sin 45}{\sin 75} = \frac{\sin 15 \cdot \sin 45}{\cos 15} = \tan 15 \sin 45$$

$$\frac{A}{B} = \frac{\frac{1}{2}PQ(x)}{\frac{1}{2}SR(y)} = \frac{x}{y} = \frac{\sin 45}{\tan 15 \sin 45} = \frac{1}{\tan 15} = \frac{1}{2-\sqrt{3}} = \underline{\mathbf{2+\sqrt{3}}}$$



F) $4d + 5k = 540$ ➔ $d = (540 - 5k)/4 = (135 - k) - k/4$

k must be a multiple of 4.

$k = 4t$ ➔ $m\angle E = 20t$

$d = m\angle A = m\angle B = m\angle C = m\angle D = (540 - 20t)/4 = 135 - 5t$

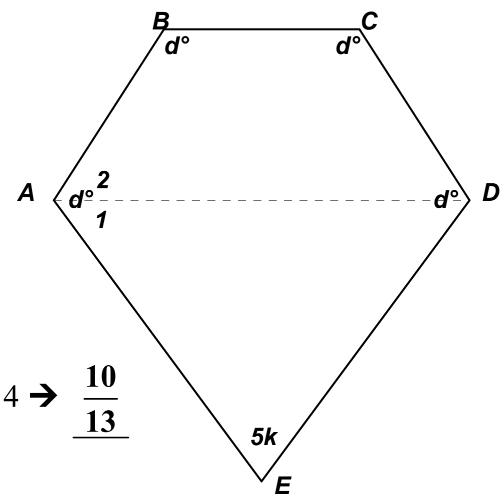
$m\angle EAD = (180 - 20t)/2 = 90 - 10t$

$m\angle DAB = (135 - 5t) - (90 - 10t) = 45 + 5t$

$$\frac{m\angle EAD}{m\angle DAB} = \frac{90-10t}{45+5t} = \frac{18-2t}{9+t}$$

Trying values of t we get the following ordered pairs:

(1, 8/5) (2, 14/11) (3, 1) …. This is a decreasing sequence and $t = 4$ ➔ $\underline{\mathbf{\dfrac{10}{13}}}$



Alternate solution #1:

$5d + 5k = 540$ ➔ $d = \dfrac{540-5k}{4} = 135 - \dfrac{5}{4}k$.

Since d must be an integer, k must be divisible by 4.

Since $m\angle E = 5k < 180$, $k < 36$. Therefore $k = 4, 8, 12, 16, \ldots, 32$.

The chart below indicates as k increases the required ratio decreases and the largest value less than 1 is highlighted.

k	***d*** **(m∠*BAE*)**	**m∠*E***	**m∠1 (∠*EAD*)**	**m∠2 (∠*DAB*)**	**Ratio**
4	130	20	80	50	8/5
8	125	40	70	55	14/11
12	120	60	60	60	1
16	115	80	50	65	**10/13**

Alternate solution #2:

**** $4d + 5k = 540$ ➔ d must be a multiple of 5. For ADE to be a triangle, $0 < 5k < 180$ or $0 < k < 36$. Substituting in ****, we have $90 < d < 135$. By trial and error (and the fact that d is a multiple of 5), we test 130, 125, 120, … . These results are contained in the table above, referencing the second column as the key field.