MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 – OCTOBER 2014
ROUND 1 VOLUME & SURFACES

ANSWERS

A) ______________________________

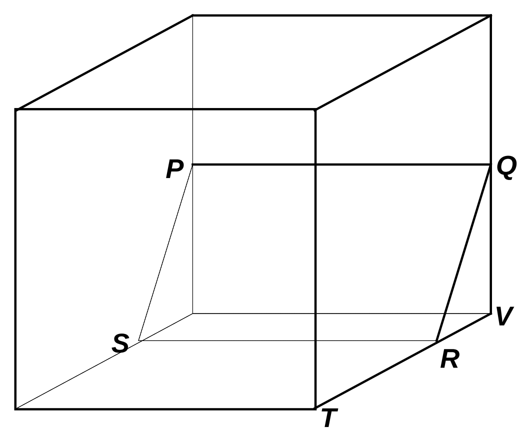
B) ______________________________

C) (____________ , ____________)

A) The volume of a cone is 18π cubic units. The circumference of its circular base is 8π units. Compute the cube root of the height of the cone.

B) A cross section of a sphere 9 units from the center of the sphere is a circle that has an area of 144π units^2. Compute the volume of a hemisphere of this sphere.

C) The sides of a cube have length 12 units. Plane $PQRS$ divides the cube into two regions whose volumes are in a ratio of 3 : 29. P and Q are each midpoints of a pair of opposite edges. R and S are located on another pair of opposite edges such that $\overline{SR} \parallel \overline{PQ}$ as shown. R divides $\overline{TV}$ into a ratio of $a : b$, where a and b are integers and $a > b$. Compute the ordered pair (a, b).



MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014
ROUND 2 PYTHAGOREAN RELATIONS IN RECTILINEAR FIGURES

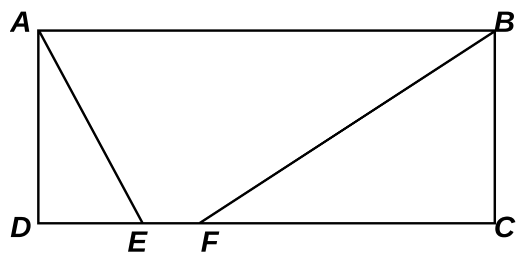
ANSWERS

A) ______________________________

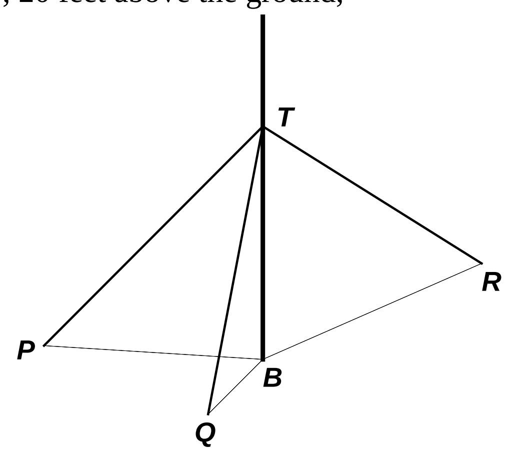
B) ______________________________

C) ______________________________

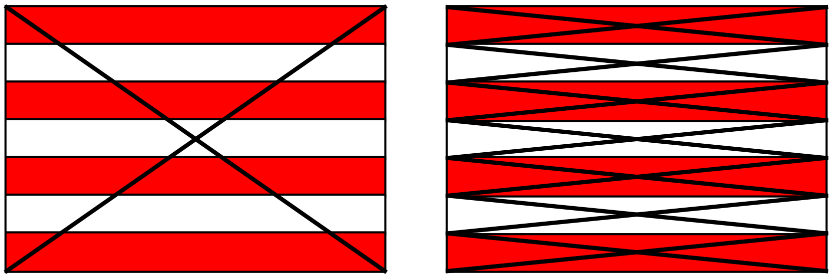
A) In rectangle *ABCD*, *AB* = 24, *AD* = 8, *AE* = 10 and *BF* = 17.
Compute the area of *ABFE*.



B) Three wires support a newly planted oak tree at a point *T*, 20 feet above the ground, and anchored at points *P*, *Q* and *R*.
BQ = 15, *BP* = 21 and *P*, *B* and *Q* are noncollinear.
If the length of $\overline{PQ}$ is an integer, what is the <u>maximum</u> perimeter of ΔTPQ?



C) A rectangular flag has exactly 7 stripes of equal width.
The flag designer was not specific when asking the manufacturer to embroider an X on the flag. The manufacturer made two models as shown in the diagrams below, where one model has one X and the other has an X on every stripe. The diagonal in the diagram on the left is 2 less than 3 times a diagonal in the diagram on the right. The total length of all the embroidered "X"s on both flags is 256 inches and the dimensions of the flags are integers.
Compute the area of the flag (in inches^2).



MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014
ROUND 3 ALG 1: LINEAR EQUATIONS

ANSWERS

A) ________________________________

B) ________________________________

C) ________________________________

A) Three pounds of beans cost \$4.80.
Two pounds of corn cost \$1.80. How many pounds of beans must be added to 30 pounds of corn to form a mixture that sells for \$1.10 per pound?

B) Consider the linear equation $y = 3x + 2$.
As x increases from k to $k + 3$, the value of y increases to $2y + 1$.
Compute this unique value of k.

C) At a scenic overlook, some artists were painting at their easels. All easels have 3 legs. All chairs have 4 legs. These artists either paint while standing, or paint while sitting. Suppose X artists stood at their easels, and Y artists sat in a chair, while using their easels. The total number of easel legs and chair legs was 92. If the total number of artists was less than 25, determine <u>all</u> possible ordered pairs (X, Y).

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014
ROUND 4 ALG 1: FRACTIONS & MIXED NUMBERS

ANSWERS

A) ______________________________

B) ______________________________

C) ______________________________

A) For some positive integer x, each term in the sum $\frac{60}{x}+\frac{60}{x+1}+\frac{60}{x+2}+\frac{60}{x+3}+\frac{60}{x+4}$ is an integer. Compute the <u>minimum</u> value of this sum.

B) It has long been a marathoner's dream to be the first to <u>break</u> the 2 hour barrier for this 26.2 mile race. Assume a marathon is <u>exactly</u> 26.2 miles and that breaking the 2 hour barrier means by at least 1 second. A runner averaging 4 minutes and 35 seconds per mile would miss breaking the 2 hour barrier by k seconds, where k is an integer. Compute k.

C) Find an equivalent simplified expression for $\dfrac{8x^2\left[4+\left(\frac{x}{2}-\frac{2}{x}\right)^2\right]}{\left(x^2+4\right)^2}$, given $x \neq 0$.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014
ROUND 5 INEQUALITIES & ABSOLUTE VALUE

ANSWERS

A) ______________________________

B) ______________________________

C) ______________________________

A) Compute the 4 values of x for which $\left|5-|x+1|\right|=2$.

B) Given: $-2 < x < 2$
Determine <u>all</u> values of x which satisfy $|2x-2|+|4x-4|=|5x+11|$

C) Solve for x.

$$\frac{6}{x-3}+\frac{1}{x-5}\geq 3$$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014
ROUND 6 ALG 1: EVALUATIONS

ANSWERS

A) ______________________

B) (__________ , __________)

C) ______________________

A) Given: $\begin{cases} w = 5 - x \\ x = 6y + 1 \\ y = -\dfrac{2}{3}z \end{cases}$

Find a simplified expression for *z* in terms of *w*.

B) Definition:

$f(x, y) = x + \dfrac{1}{y}$, that is, the sum of the first argument and the reciprocal of the second argument.

For example, $f(4,2) = 4.5$, $f(2,4) = 2.25$.

If $\dfrac{f(x, y)}{f(y, x)} = \dfrac{2}{3}$ and $x - y = 4$, find the ordered pair (x, y).

C) Given: $x = 5x - 2$, $\boxed{x} = \dfrac{x+a}{b}$

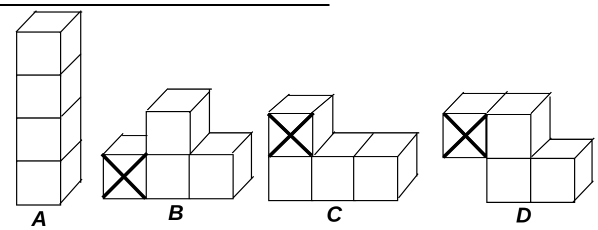
If *a* and *b* are positive integers, where $a < 10$ and $b < 10$, determine the largest possible value of $a + b$ if $\boxed{x} = \boxed{x}$.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014
ROUND 7 TEAM QUESTIONS
ANSWERS

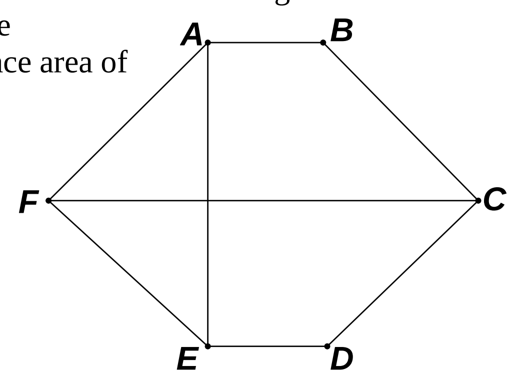
A) ____________________ D) (__________ , __________)

B) ____________________ E) ____________________

C) ____________________ F) ____________________



A) Four unit blocks (1 x 1 x 1) are glued together face-to-face in a single layer to form different geometric solids. This may be done in 4 different ways. Multiple copies of each solid are available. Copies of each solid are to be placed in all possible <u>stable</u> positions with a <u>different</u> number of surfaces in contact with the table top.
Assume that any solid with 1, 2, 3 or 4 faces, or exactly 2 edges in contact with the table, is in a stable position. In some copies of *B*, *C* and *D*, the cubes marked with an X are weighted. Only glued faces and faces that lie in the plane of the table top are considered to be not exposed. The <u>sum</u> of the total exposed surface area of all these solids is *K*. Compute *K*.



B) In hexagon *ABCDEF*, *AB* = *DE* and
BC = *CD* = *EF* = *FA* = 2·*AB*
A is the reflection of *E* and *B* is the reflection of *D* across $\overleftrightarrow{FC}$
If *FC* = *AE* + 1 and the area of Δ*AFE* is 2.875, compute *AB*.

C) Given: $\begin{cases} x+ay=b^2 \\ x-by=a^2 \end{cases}$ for positive integers *x*, *y*, *a* and *b*.
If $a:b=4:7$, compute the <u>minimum</u> sum of $x+y$.

D) Let *x* be the smallest integer for which $y=\frac{7}{x+4}-\frac{3}{x-3}>0$. Determine the ordered pair (*x*, *y*).

E) For *n* = 1, $\frac{n^3-32}{n^2+30}$ is an integer. Determine the <u>largest</u> <u>even</u> positive integer for which $\frac{n^3-32}{n^2+30}$ is also an integer.

F) A 10 gallon container *A* is full of water and containers *B* and *C*, with 7 gallon and 4 gallon capacities respectively, are empty. Water can only be transferred from one container to another if one of the following conditions is satisfied:
- the container into which water is poured is completely filled
- the container from which the water is poured is completely emptied

Let the initial state be $(A,B,C)=(10,0,0)$. After the first transfer, we would have $(3,7,0)$ or $(6,0,4)$. After a total of *k* transfers, I can get $(5,5,0)$. Compute the <u>minimum</u> possible value of *k*.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014 ANSWERS

Round 1 Geometry Volumes and Surfaces

A) $\frac{3}{2}$ B) 2250π C) $(5,3)$

Round 2 Pythagorean Relations

A) 108 B) 89 C) 420

Round 3 Linear Equations

A) 12 B) 2 C) (19, 5), (12, 8), (5, 11)

Round 4 Fraction & Mixed numbers

A) 87 B) 6 C) 2

Round 5 Absolute Value & Inequalities

A) $-8, -4, 2, 6$ (in any order) B) $-\frac{5}{11}$ C) $3 < x \le \frac{13}{3},\ 5 < x \le 6$

Round 6 Evaluations

A) $\frac{w-4}{4}$ (or equivalent) B) $(-8, -12)$ C) 13

Team Round

A) 240

B) $\frac{6}{5}$ (or 1.2)

C) 40

D) $(-3, 7.5)$ (or equivalent)

E) 14

F) 8

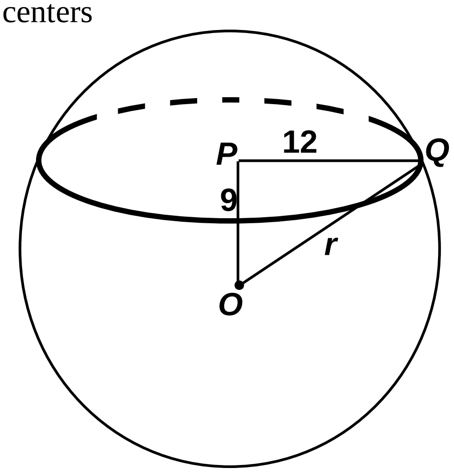
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014 SOLUTION KEY

Round 1

A) $C = 2\pi r = 8\pi \Rightarrow r = 4 \Rightarrow \text{area}_{\text{base}} = 16\pi$

$$V = \frac{1}{3}\pi r^2 h \Rightarrow 18\pi = \frac{16}{3}\pi h \Rightarrow h = \frac{54\pi}{16\pi} = \frac{27}{8} \Rightarrow \sqrt[3]{\frac{27}{8}} = \underline{\mathbf{\frac{3}{2}}}$$ (or **1.5**)

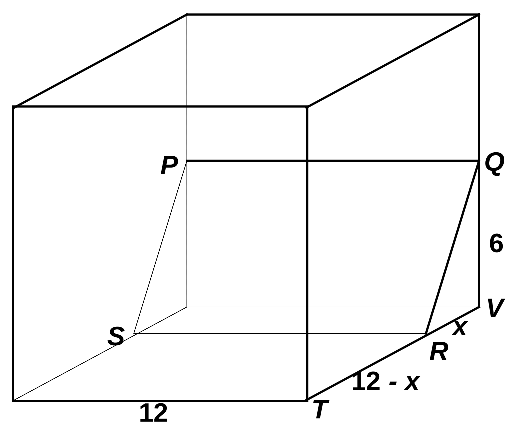
B) Let r denote the radius of the sphere and O and P be the centers of the sphere and the cross section respectively.



The radius of the cross section is 12. Since $\overline{OP}$ is perpendicular to the cross section, ΔPOQ is a right triangle and $r = OQ = 15$.
The volume of the hemisphere is

$$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi(15)^3 = 10(15)^2\pi = \underline{\mathbf{2250\pi}}$$

C) The volume of the region "behind" rectangle $PQRS$ is $\frac{1}{2}\cdot 6x\cdot 12 = 36x$. The given ratio is

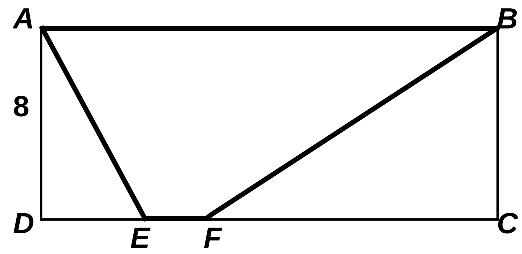


$$\frac{36x}{12^3 - 36x} = \frac{x}{48 - x} = \frac{3}{29} \Rightarrow 29x = 144 - 3x \Rightarrow x = \frac{144}{32} = \frac{9}{2}$$

Thus, $\frac{TR}{RV} = \frac{12 - 4.5}{4.5} = \frac{7.5}{4.5} = \frac{5}{3} \Rightarrow (a,b) = \underline{(\mathbf{5,3})}$.

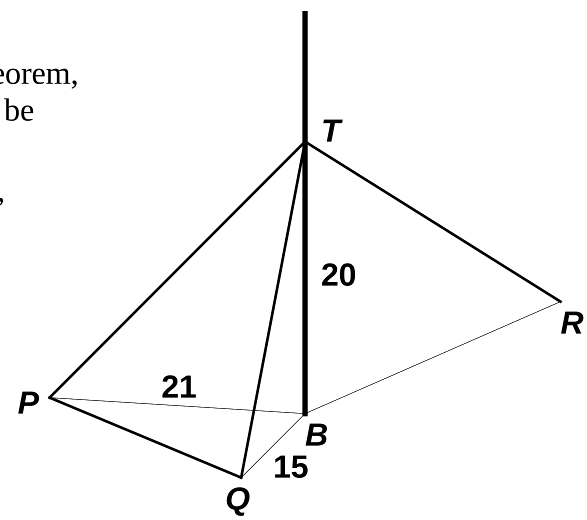
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014 SOLUTION KEY

Round 2



A) $DE = 6, FC = 15, AB = 24 \Rightarrow EF = 3$

Thus, the area of $ABFE$ is $\frac{1}{2} \cdot 8 \cdot (3 + 24) = \underline{\mathbf{108}}$



B) Using special right triangles or the Pythagorean Theorem, PT = 29 and QT = 25. The triangle inequality must be satisfied in both ΔTPQ and ΔBPQ.

In ΔTPQ, PQ < 29 + 25 = 54; however, in ΔBPQ,

PQ < 21 + 15 = 36.

Thus, the maximum integer value of PQ is 35

$\Rightarrow$ the maximum perimeter of ΔTPQ is **<u>89</u>**.

C) Let a denote the width of a stripe and b its length.

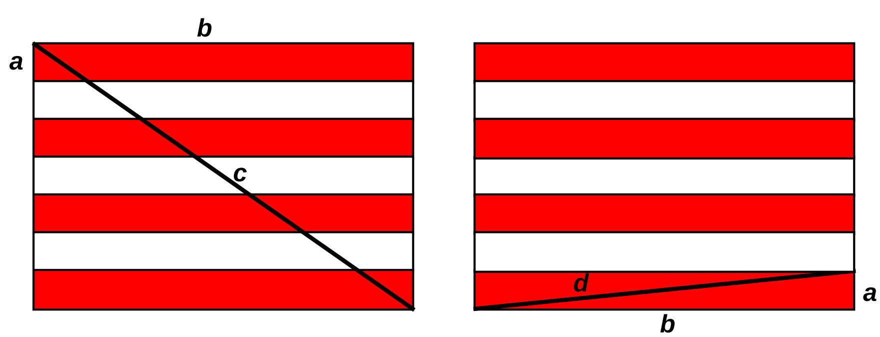
Let c and d denote the length of the diagonals on each flag.

Then: $\begin{cases} (7a)^2 + b^2 = c^2 \text{ (flag on left)} \\ a^2 + b^2 = d^2 \quad \text{(flag on right)} \end{cases}$ and $\begin{cases} 2c + 14d = 256 \\ c = 3d - 2 \end{cases}$

Substituting in the second pair of equations, $(3d - 2) + 7d = 128 \Rightarrow d = 13, c = 37$

$d = 13 \Rightarrow (5,12,13)$, $c = 37 \Rightarrow (35,12,37)$

and the dimensions of the flag are 12 x 35 producing an area of **<u>420</u>**.



MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014 SOLUTION KEY

Round 3

A) The cost per pound of the beans is \$1.60, while the corn costs \$.90 per pound.
Let x denote the # pounds of beans needed. Then:
$1.60x + 0.90(30) = 1.10(30 + x) \Rightarrow 160x + 2700 = 3300 + 110x \Rightarrow 50x = 600 \Rightarrow x = \underline{\mathbf{12}}$

B) For $x = k$, $y = 3k + 2$
For $x = k + 3$, $y = 3(k + 3) + 2 = 3k + 11$
Thus, $3k + 11 = 2(3k + 2) + 1 \Rightarrow 3k = 6 \Rightarrow k = \underline{\mathbf{2}}$
Check: For x = 2, y = 8. For x = 5, y = 17. $(17 = 2 \cdot 8 + 1)$

C) $3X + 7Y = 92 \Rightarrow X = \dfrac{92 - 7Y}{3} = 30 - 2Y + \dfrac{2 - Y}{3}$ Clearly, our choices for Y are restricted to 2, 5, 8, 11 …, so that the last term is an integer. Thus, we have ordered pairs (X, Y) = (26, 2), (19,5), (12, 8), (5, 11).
Note that since the slope of the line $3X + 7Y = 92$ is $\dfrac{-3}{7}$ or $\dfrac{3}{-7}$, as X decreases by 7, Y increases by 3.
Therefore, we look no further. (The next ordered pair would be (–2, 14).)
Since $X + Y < 25$, the possible ordered pairs are **<u>(19, 5)</u>**, **<u>(12, 8)</u>** and **<u>(5, 11)</u>**.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014 SOLUTION KEY

Round 4

A) We require that 60 be divisible by 5 consecutive integers.
This is only true for 1 …5 and 2 … 6. The latter gives the smaller sum.
For x = 2, we have $30+20+15+12+10=\underline{\mathbf{87}}$.

B) The 2 hour barrier is equivalent to $2\cdot 60\cdot 60=7200$ seconds. Our future marathoner must complete the course in 7199 seconds. At 4:35 per mile, (s)he would take $26.2(275)=7205$ seconds.
Thus, k = **6**.
Note:
Meb Keflezighi (pronounced Kef-lez-ghee) won the 2014 Boston Marathon in 2:08:37, beating the Kenyan Wilson Chebet by 11 seconds. Meb averaged approximately 4 minutes 54.5 seconds per mile. The marathon is actually 26 miles 385 yards (or 26.21875 miles).

C) $\dfrac{8x^2\left[4+\left(\dfrac{x}{2}-\dfrac{2}{x}\right)^2\right]}{\left(x^2+4\right)^2}\Leftrightarrow\dfrac{8x^2\left[4+\dfrac{x^2}{4}-2+\dfrac{4}{x^2}\right]}{x^4+8x^2+16}\Leftrightarrow\dfrac{16x^2+2x^4+32}{x^4+8x^2+16}\Leftrightarrow\dfrac{2\left(\cancel{x^4+8x^2+16}\right)}{\cancel{x^4+8x^2+16}}=\underline{\mathbf{2}}$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014 SOLUTION KEY

Round 5

A) $\left|5-|x+1|\right| = 2 \Leftrightarrow 5-|x+1| = \pm 2 \Leftrightarrow |x+1| = 3, 7$

Therefore, $x+1=\pm 3$ or $x+1=\pm 7$ and we have our 4 solutions: **<u>2, $^{-}$4, 6, $^{-}$8</u>** (in any order)

B) The equation simplifies to $2|x-1|+4|x-1| = 6|x-1| = |5x+11|$

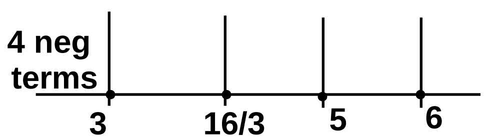
Over the stated domain $-2 < x < 2$, the original equation simplifies to $6|x-1| = 5x+11$.

$$\Rightarrow\ 5x+11 = \begin{cases} 6(x-1) \text{ if } x \ge 1 \\ 6(1-x) \text{ if } x < 1 \end{cases} \Rightarrow x = \begin{cases} 17 \\ -\dfrac{5}{11} \end{cases}$$

The only value in the stated domain is $\underline{\boldsymbol{-\dfrac{5}{11}}}$.

C) $\dfrac{6(x-5)+x-3-3(x-3)(x-5)}{(x-3)(x-5)} \ge 0 \Rightarrow \dfrac{7x-33-3x^2+24x-45}{(x-3)(x-5)} \ge 0$

$\Rightarrow \dfrac{3x^2-31x+78}{(x-3)(x-5)} \le 0 \Rightarrow \dfrac{(3x-13)(x-6)}{(x-3)(x-5)} \le 0$



The critical points 3, 13/3, 5 and 6 divide the number line into 5 regions.
At the extreme left (x < 3) all four terms are negative and, moving to the right, each time a boundary is crossed, one less term is negative. Thus, the regions with a negative quotient are $3 < x < 13/3$ and $5 < x < 6$ and equality occurs at x = 13/3 and 6 $\Rightarrow$ $\underline{\boldsymbol{3 < x \le \dfrac{13}{3},\ 5 < x \le 6}}$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014 SOLUTION KEY

Round 6

A) $\begin{cases} (1)\ \ w = 5 - x \\ (2)\ \ x = 6y + 1 \ \text{ Substituting for } x \text{ in (1), } w = 5 - (6y+1) \Rightarrow (4)\ \ w = -6y + 4 \\ (3)\ \ y = -\frac{2}{3}z \end{cases}$

Substituting in (4) for y, $w = -6\left(-\frac{2}{3}z\right) + 4 = 4z + 4 \Rightarrow z = \underline{\boldsymbol{\frac{w-4}{4}}}$ (or equivalent).

B) $\dfrac{f(x,y)}{f(y,x)} = \dfrac{x + \frac{1}{y}}{y + \frac{1}{x}} = \dfrac{\frac{xy+1}{y}}{\frac{xy+1}{x}} = \dfrac{x}{y} = \dfrac{2}{3} \Rightarrow 3x = 2y$

$\begin{cases} x - y = 4 \\ 3x = 2y \end{cases} \Rightarrow 3x - 3y = 12 \Rightarrow 2y - 3y = 12 \Rightarrow y = -12 \Rightarrow (x, y) =$ **<u>(–8, –12)</u>**

C) Given: $x = 5x - 2$, $\boxed{x} = \dfrac{x+a}{b}$

$\boxed{x} = \boxed{x} \Leftrightarrow 5\left(\dfrac{x+a}{b}\right) - 2 = \dfrac{5x - 2 + a}{b}$

$\Leftrightarrow \dfrac{5x + 5a - 2b}{b} = \dfrac{5x - 2 + a}{b}$

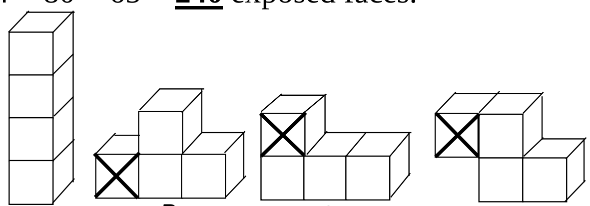
For nonzero values of b, this is only true if $5a - 2b = -2 + a \Leftrightarrow 4a = 2b - 2 \Leftrightarrow b = 2a + 1$.
To maximize $a + b$, we take the maximum value of b. $b = 2a + 1 = 9 \Rightarrow a = 4 \Rightarrow a + b = \underline{\mathbf{13}}$.

Check: For $(a,b) = (4,9)$, both expressions evaluate to $\dfrac{5x+2}{9}$.

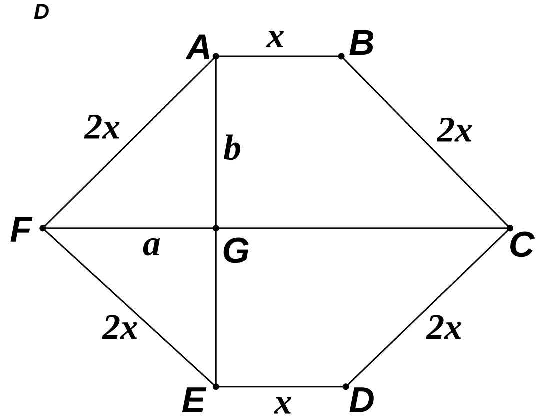
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014 SOLUTION KEY

Team Round

A) Four blocks have a total of 24 faces, but, after gluing, 6 exposed faces are lost. Thus, the maximum number of exposed faces is 18, if a stable position with exactly 2 edges in contact with the table top exists (which is the case for *B*, *C* and *D*).
A is stable resting on 1 or 4 faces, resulting in 17 or 14 exposed faces.
B is stable resting on 0, 1, 3 or 4 faces, resulting in 18, 17, 15 or 14 exposed faces.
C is stable resting on 0, 1, 2, 3 or 4 faces, resulting in 18, 17, 16, 15 or 14 exposed faces.
D is stable resting on 0, 1, 2 or 4 faces, resulting in 18, 17, 16, or 14 exposed faces.
Thus, we have 31 + 64 + 80 + 65 = **240** exposed faces.



B) Let $AB = DE = x$, $BC = CD = EF = FA = 2x$
Let G denote the intersection of $\overline{AE}$ and $\overline{FC}$
Let $FG = a$ and $AG = b$. Then:
$FC = AE + 1 \Rightarrow$ (1) $x + 2a = 2b + 1$
(2) $a^2 + b^2 = 4x^2$
(3) $area(\Delta AFE) = ab = 2.875 = \frac{23}{8}$



(1) $\Rightarrow \frac{x-1}{2} = b - a \Rightarrow (x-1)^2 = 4(a^2+b^2) - 8ab$ (***)
Substituting, using (2) and (3) in (***),
$(x-1)^2 = 4(4x^2) - 23 \Leftrightarrow 15x^2 + 2x - 24 = (5x-6)(3x+4) = 0$

Thus, $AB = x = \underline{\mathbf{\frac{6}{5}}}$ (or **1.2**).

C) Since $a:b = 4:7$, let $a = 4c,\ b = 7c$. Since $\begin{cases} x + ay = b^2 \\ x - by = a^2 \end{cases}$, after subtracting, we have

$(a+b)y = b^2 - a^2 = (b+a)(b-a) \Rightarrow y = b - a = 3c$ [$a + b \neq 0$]
Substituting in the first equation,
$x + a(b-a) = b^2 \Rightarrow x = a^2 + b^2 - ab = 16c^2 + 49c^2 - 28c^2 = 37c^2$.

Thus, $\frac{x}{y} = \frac{37c^2}{3c} \Rightarrow x = \frac{37c}{3}y$ (Recall: *x*, *y* and *c* are all positive integers.)

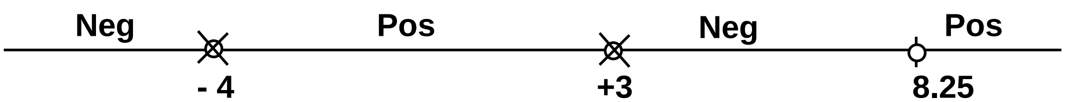
To minimize both *x* and *y*, we take $c = 1 \Rightarrow a = 4$, $b = 3$ and $x = \frac{37}{3}y \Rightarrow$

$(x, y) = (37, 3) \Rightarrow x + y = \underline{\mathbf{40}}$. Check: $37 + 4 \cdot 3 = 49 = 7^2$, $37 - 7 \cdot 3 = 16 = 4^2$

Team Round - continued

D) $\dfrac{7}{x+4}-\dfrac{3}{x-3} = \dfrac{7(x-3)-3(x+4)}{(x-3)(x+4)} = \dfrac{(4x-33)}{(x-3)(x+4)}$

The sign of the difference equals the sign of this equivalent quotient.
Whereas the sign of a difference can be hard to determine, the sign of a quotient is easy to determine, especially when both the numerator and denominators have been factored.
We simply count the number of negative factors!
As x increases, each of the parenthesized expressions increases and becomes positive.
The critical values are -4, 3 and 33/4 (= 8.25).



At the extreme left, all three expressions are negative (hence the quotient is also).
At the extreme right, all three expressions are positive (hence the quotient is also).

Testing between -4 and 3, of the three expressions only $(x + 4)$ becomes positive and the sign of the quotient is determined by two negatives and one positive; hence the quotient is positive.

Testing between 3 and 8.25, of the three expressions only $(4x - 33)$ remains negative and the sign of the quotient is determined by one negative and two positives; hence the quotient is negative.

Thus, as x increases from left to right along the number line, the sign of the quotient changes as we pass each critical point. In this case NEG – POS – NEG – POS.
Thus, $x = -3$ and substituting $y = \dfrac{4(-3)-33}{(-3-3)(-3+4)} = \dfrac{-45}{-6} = \dfrac{15}{2}$ or 7.5 $\Rightarrow$ **<u>(–3, 7.5)</u>** or equivalent

E) By long division, $\dfrac{n^3-32}{n^2+30} = n-2\left(\dfrac{15n+16}{n^2+30}\right)$
Since we know n is even, let $n = 2k$, where k is an integer.
Then $\left(\dfrac{15n+16}{n^2+30}\right) = \left(\dfrac{30k+16}{4k^2+30}\right) = \dfrac{15k+8}{2k^2+15}$. For this last fraction to be an integer, the numerator must be greater than or equal to the denominator.
$15k+8 \geq 2k^2+15 \Rightarrow 2k^2-15k+7 \leq 0 \Rightarrow (2k-1)(k-7) \leq 0 \Rightarrow \dfrac{1}{2} \leq k \leq 7 \Rightarrow n = 2, 4, ..., 14$.
Since we are looking for the *largest* possible even integer, we now resort to brute force, starting with $k = 7 \Rightarrow \dfrac{105+8}{98+15} = 1$. Bingo! Thus, n = **<u>14</u>**.
(In fact, 14 is the only even value of n, for which the given quotient is an integer.)

Team Round – continued

F) The following sequence shows how $(5,5,0)$ can be obtained, starting with a first transfer to $(6,0,4)$.

$(6,0,4) \Rightarrow (6,4,0) \Rightarrow (2,4,4) \Rightarrow (2,7,1) \Rightarrow (9,0,1) \Rightarrow (9,1,0) \Rightarrow (5,1,4) \Rightarrow (5,5,0)$

This required 8 transfers.

Convince yourself that, starting with $(6,0,4)$, no shorter sequence is possible.

Starting with $(3,7,0)$ produces the following tree which eventually produces $(6,0,4)$, and, consequently, a longer sequence.

$(3,7,0)$

$(0,7,3)$ $(3,3,4)$

$(0,6,4)$ $(7,0,3)$ $(0,6,4)$ $(6,0,4)$

$(6,0,4)$ $(6,0,4)$ $(6,0,4)$

Thus, the minimum sequence is **8** transfers.